Mathematics Lesson Plan for Seventh Grade

For the Lesson on Friday, January 24, 2003
At the Harriet Tubman Middle School, Portland, Oregon
Instructor: Akihiko Takahashi

1. Title of the Lesson: Which container holds a larger amount of popcorn?

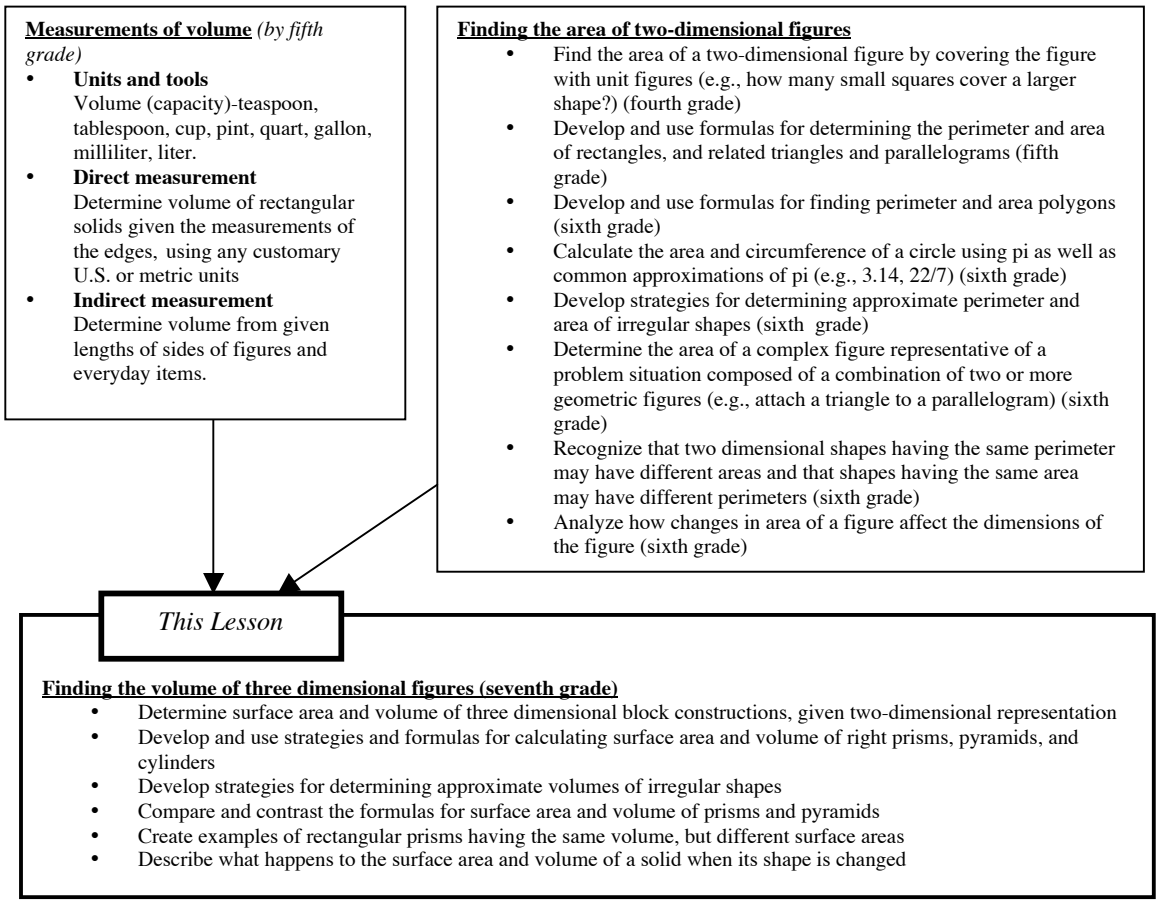
2. Goal:

   a. To deepen students' understanding of the concept of volume through problem solving

   b. To help students recall the way to find the volume of a rectangular container

   c. To help students recognize that the volume of a rectangular prism can be found by multiplying the area of the base and height so that they will be able to extend this idea to find the volume of other right prisms and cylinders

3. Relationship of the Lesson in the Oregon Grade-level Standards, Mathematics



4. Instruction of the Lesson

The Oregon Standards expect students to understand the formula for finding the volume of right prisms, pyramids, and cylinders in seventh grade. In order to fully understand this formula so that they can use it as a tool to solve various problems, students should have opportunities to develop a conceptual understanding of the formula by incorporating their previously learned knowledge, rather than simply memorizing the formula that was given by a teacher.

Prior to the seventh grade, students learn that volume (capacity) can be measured by using standard and non-standard units. For example, the volume of a rectangular container can be measured by filling it up with unit cubes. Through this concrete experience, students also develop the formula, $V = length \times width \times height$, for finding the volume. At the same time, students learn the concept of area and develop various formulas for finding the area of various polygons and a circle.

In order to build upon the students' previously learned knowledge mentioned above, this lesson is designed as an introduction of a series of lessons designed to help them to conceptually understand the formula for finding the volume of a right prism (e.g., triangular prism and pentagonal prism) and a cylinder. To find the volume of right prisms and a cylinder, students need to extend their previously learned knowledge for finding the volume of a rectangular solid, $V = length \times width \times height$, to the more generalized formula of $V = area\ of\ base \times height$.

Although in the formula $V = area\ of\ base \times height$ it looks like you are multiplying an area and a length to find the volume, the idea behind this formula is finding the number of unit cubes that can be put in a prism. The first step to understand the idea is finding the number of unit cubes that can be put in one layer on the bottom of the prism, then multiplying the number of unit cubes in the bottom layer (*area of base*) by the number of layers (*height*). The number of stacked layers is equal to the height of the prism. In the case of a rectangular prism, the area of the base can be measured by $length \times width$ (Filling and Wrapping, Teachers guide).

If the students can understand the formulas for finding the areas of various figures that they have learned prior to the seventh grade, they can incorporate them to find the volume of any polygonal prism or a cylinder using the formula $V = area\ of\ base \times height$.

As an introduction, this lesson is designed to help students develop the formula for finding the volume of a rectangular prism, $V = area\ of\ base \times height$, by using their previous experience with the formula $V = length \times width \times height$, through a concrete activity.

To provide students with an opportunity to extend their previous knowledge, a problem from the Connected Mathematics 'Filling and Wrapping' is chosen and revised. In the 'Filling and Wrapping', the original problem is given as an extension. This original asks students to compare the volumes of two cylinders made by the same size sheet of paper. To give students this problem as an introduction, the problem in this lesson plan is revised from the original problem. The problem for this lesson asks students to compare the volume of two rectangular prisms made by the same size sheet of paper. The problem situation is also revised to one that might be more familiar to students' daily lives.

Unlike the typical flow of structured problem solving, the lesson begins with a hands-on demonstration in order to see the solution first. Then, they are going to have an opportunity to solve the problem by calculation using the formula that they learned previously. The reasons for this decision are: (1) the instructor does not know the students previous learning experience, (2) the lesson is held in different circumstances from the one that their regular classroom has, and (3) this is the first experience students have had to expose their learning process to many observers. Therefore, the lesson does not include an individual or group hands on activity at the beginning. If this lesson were taught in the students' regular mathematics class, it might be a good idea to include a hands-on experiment.

## 5. Lesson Procedure

| Learning Activities<br>Teacher's Questions and Expected Students' Reactions | Teacher's Support | Points of Evaluation |
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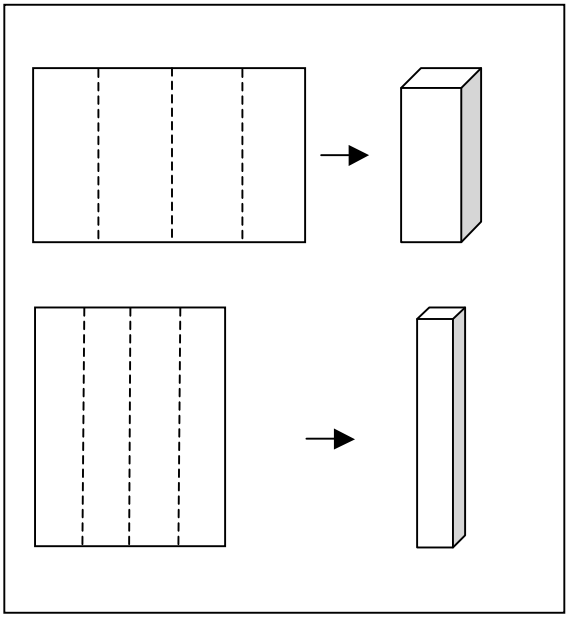
| **1. Introduction** | | |
| You are standing in the concession line at the movie theatre. You and your friends decide to share a container of popcorn. One of your friends insists on getting the shorter, stouter container. Both containers cost the same amount of money. When you get to the front of the line, the server tells you that it doesn't matter; they are both the same size container. He explains that both containers were created from the same size sheet of paper. The paper was just used differently. | Tell students the problem as a story, and then, pose the problem in written format on the blackboard | Do the students understand how two different containers were created from the same size sheet of paper? |
|  | Show students the actual size containers created from the same size sheet of paper.<br>• Demonstrate two different ways to create a container by using the same size sheet of paper if necessary. | |
| 1) Do both containers have the same amount of popcorn? If not, which container has a larger amount of popcorn? Why do you think so?<br>• Both have the same amount<br>• The shorter, stouter container has more<br>• The taller, skinnier container has more | Encourage students to talk freely about their ideas for solving the problem | Did each student write his/her prediction on their worksheet? |
| 2) Make sure which container has more by comparing the amount of popcorn in the two containers | Ask students how to compare the amount of popcorn in two containers | Can each student see which container has the greater amount of popcorn? |

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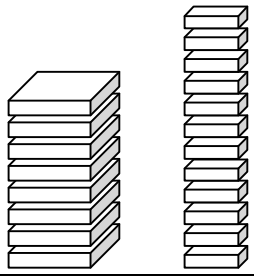
| **2. Posing Problem**<br>Why do two containers have different amounts of popcorn even though both are created from the same size sheet of paper?<br>Find out the volume of each container and compare them.<br><br>8 inches<br>  <br>12 inches<br><br>Students' anticipated solutions<br>• By using the formula, *length* × *width* × *height*, calculate the volume of each container<br>The shorter, stouter container $3 \times 3 \times 8 = 72 \text{in}^3$<br>The taller, skinnier container $2 \times 2 \times 12 = 48 \text{in}^3$ | To find out the volume of two containers, the lengths of each side of the paper are given. These numbers are chosen to help students find the volume easily.<br><br>In case students cannot recall a way to find the volume of rectangular solid, a unit-cube model with one-inch sides is prepared. | Do students recall a way to find the volume of a rectangular solid given the measurements of the edges? |
| **3. Discussion**<br>(1) Ask students to explain their solutions. Students are expected to explain by using their previous knowledge.<br>(2) Help students understand the number of unit cubes that can be put in one layer on the bottom of the prism multiplied by the number of layers. The number of layers is equal to height of the prism. In the case of a rectangular prism, the area of the base can be measured by *length* × *width*.<br>The shorter, stouter container has one layer with 9 $\text{in}^3$ $(3 \times 3)$. The number of stacked layers is 8<br>The taller, skinnier container has one layer with 4 $\text{in}^3$ $(2 \times 2)$. The number of stacked layers is 12<br>Although the skinnier container is taller, each layer is less than half of the stouter container's one<br>(3) The formula for finding the volume of a rectangular container, *length* × *width* × *height*, can also be summarized as *area of base* × *height* | Students are encouraged to use the popcorn containers to explain what the formula means and how they can measure the volume of the containers by using the formula<br><br>Help students conceptualize the idea of 'stacking a layer in the base by showing the following figure if necessary.<br><br>  | Do students understand how the formula to find the volume of the containers works? |
| **4. Summing up**<br>(1) Using the writing on the blackboard, review what students learned through the lesson.<br>(2) Ask students to write a journal entry about what they learned through the lesson. | | |

6. Evaluation
   a. Were the students able to recall a way to find the volume of a rectangular container?
   b. Were students able to extend their previous knowledge to recognize that the volume of a rectangular prism can be found by multiplying the area of the base by the height?