Mathematics Lesson Plan for 3rd Grade—Decimals

For the lesson on May 9, 2008
at the Chicago Lesson Study Conference

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1. **Title of the Lesson:** *Where Should You Turn?*

2. **Goals of the Lesson:**
   a. Students will understand that there are numbers between the whole numbers, and that those numbers can be expressed using decimals;
   b. Students will gain an approximate sense of the positioning of decimal numbers (tenths) on a linear scale;
   c. Students will begin to understand that decimal numbers are a natural extension of the base-10 numeration system.
   d. Provide opportunities for students to recognize the importance of working with their peers in order to deepen their understanding of mathematics.

3. **Relationship of the Lesson to the Standards**

From the Illinois Assessment Framework for 3rd grade:
6.3.06 Order and compare decimals expressed using monetary units.
6.3.10 Solve problems involving the value of a collection of bills and coins whole total value is $10.00 or less, and make change.

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This Lesson

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From the Illinois Assessment Framework for 4th grade:
6.4.06 Order and compare decimals through hundredths.
Japanese Course of Study for 3rd grade:
Meaning of decimal numbers…It is also important to represent decimal numbers on the same number line where whole numbers are placed to deepen the understanding of numbers.

4. Unit Plan

Lesson	No. of days	Description
1	2	Students explore how decimals are used in real life and begin to see how they behave. Contexts include: weights on an electronic delicatessen scale, body temperature, a gas pump displaying cost and quantity, time measured by a stop watch, and driving directions from an internet site.
2	**1**	(RESEARCH LESSON) Students discover how decimal values can be used to express distances in between whole numbers of miles. They also learn the terms *decimal number, whole number,* and *decimal point.*
3	1	Students place decimals on a number line. They also learn to use "first decimal place" to refer to the tenth place.
4	1	Students learn how decimals can be used for linear measurement (in cm).
5	1	Students learn how decimals can be used for liquid measurement.
6	1	Students investigate the number 1.8 and consider it from different perspectives. In particular, they learn that 1.8 can be decomposed either into 1 and 0.8, or into eighteen 0.1s. They also compare decimals (using < and >) and compare decimals to whole numbers.
7	3	Addition of decimals: (a) with sum less than one; (b) with sum equal to one; (c) with sum greater than one (e.g. 0.8+0.4).
8	3	Subtraction of decimals: (a) less than one (0.6 – 0.2); (b) from one (1 – 0.3); (c) across one (1.4 – 0.6).
9	2	Practice.
10	1	Assessment.

5. Considerations in Planning the Unit and Lesson

The research goals of the lesson are to explore how to introduce decimals in the 3rd grade (a) through a problem-solving approach, (b) without assuming prior work with fractions, and (c) without focusing on money.

As part of our *kyozaikenkyu* we examined the following curricula:

- *Everyday Mathematics,* 3rd Edition
- *Math Trailblazers,* 2nd Edition
- *My Pals are Here!*, a new Singapore mathematics program from Marshall Cavendish.
- Tokyo Shoseki's *Mathematics for Elementary School.*

Everyday Mathematics (*EDM*) and *Math Trailblazers* (*MTB*) introduce decimal notation without attempting to motivate it; both make use of base-10 blocks to build an understanding of decimal values as fractions (where the 100-block is considered a unit, the 10-rod is 1/10, and the small cube is 1/100). Both curricula then connect this representation to money. *EDM* relies heavily on money; before the formal introduction of decimal notation there is a sequence of lessons using the calculator to explore the decimal representation of money. *MTB* makes a more cursory link,

noting that money values can be represented using base-10 blocks, then applies decimals to measurement in centimeters.

The Singapore mathematics program, which introduces decimals in 4th grade, takes a similar approach with base-10 blocks except that the notation is immediately extended to thousandths.

The Japanese Course of Study calls for both fractions and decimals to be introduced in the 3rd grade. Even though decimal numbers are listed in the text before fractions, it does not dictate which should come first. The Tokyo Shoseki text introduces decimals before fractions; it uses the context of the number line and contexts of measurement of length and capacity. (Decimals are not used in Japan with money, since the yen is the smallest currency.)

We decided to design our unit on decimals assuming no prior work with fractions. We know that in many countries it is common to introduce decimals before fractions, and we want to explore the ramifications of this ordering. One possible reason for putting decimals first is that decimals are in many ways simpler than fractions. Each point on the number line has a unique decimal representation (ignoring trailing zeroes); this is not true with fractions. And the way decimals behave is consistent with what students have experienced with whole numbers. So decimals can be considered a natural extension of the whole number system with which students are already familiar.

Even if fractions have been introduced first, it may still be beneficial to introduce decimals without initially linking them to fractions. The fraction concept may not yet be solid in the student's mind, in which case it would be imprudent to attempt to build another concept on top of it. In the long run it will be important for students to connect these concepts, but we do not believe it needs to happen in the 3rd grade.

Regarding the use of money as a context for learning fractions, *EDM* and *MTB* are far from being the only curricula to use money, and we have often heard other teachers say that they use money. The sole mention of decimals in the Illinois State Assessment Framework for 3rd grade says that students should be able to "[o]rder and compare decimals expressed using monetary units."

We believe, however, that money is in fact a bad context for building an understanding of decimals, for four reasons.

- **The language of money is idiosyncratic.** Monetary amounts are described in terms of dollars and cents, not decimal values of dollars. For instance, $1.23 is read "one dollar, 23 cents" rather than "one point two three dollars". This leads students to view the decimal point as a separator between dollars and cents and to make the predictable error of interpreting a number like 0.4 as 4 cents (we observed this with some 7th graders recently).
- **Monetary amounts are usually fixed at two decimal places.** This contributes to the error just mentioned; in general, analogies to money are unhelpful for understanding decimal values with only one decimal place or with more than two decimal places.

- **The physical structure of money does not reflect its value.** The dime is not ten times larger than a penny, the dollar is not ten times larger than the dime. The value of money is an abstraction, and despite its familiarity we find that students do not necessarily understand it.
- **Focusing on money may make it harder to create a generalized understanding of decimal numbers.** In particular, students need to understand how decimals fit into the number line and how decimals can be used in measurement.

6. Instruction of the Lesson

Although students have seen decimals, they may not have thought about what the decimal numbers mean. In the context of money students understand the decimal point only as a separator between dollars and cents (see the discussion above). The first two lessons of this unit are probably the first time most students are asked to focus their attention on decimal numbers in other contexts. For the research lesson, we decided to use the context of driving directions because measurement of linear distance abstracts easily to the number line, and shows how decimal numbers relate to whole numbers.

We want students to see that decimal numbers are a natural extension of the whole number system in which each place moving to the right is one tenth of the previous place. But typically, most work with place value goes the other way, i.e. viewing each place moving to the left as ten times the previous place.

To help students think in terms going from larger to smaller units, we provide students in this lesson with a sequence of three problems involving driving directions, at progressively smaller scales. The first problem uses a map at a scale of 100s of miles, with the directions specified to the nearest 10 miles. The second problem uses a map at a scale of 10s of miles, with directions specified to the nearest mile. And the third problem uses a map at a scale of 1 mile, with directions specified to the tenth. On all three maps, students will need to find a way to estimate the location of a certain distance.

In the previous lesson, students had several opportunities to see how decimal numbers count up and then "roll over" in much the same way that whole numbers do (e.g. 1.9 → 2.0). These experiences will help students understand that just as there are ten intervals of 10 from 100 to 200, and ten intervals of 1 from 10 to 20, there are ten intervals from 1 to 2, and those intervals can be represented with a decimal point and another digit.

We are *not* expecting students at this point to articulate a one-tenth relationship.

7. Plan of the Lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
1. Introduction Remind students of their work in the previous lesson, including directions off the internet. Driving directions contain numbers like "1.2 miles" to tell how far to drive on each road. "Today we are going to look at three different maps with driving directions, as a way to understand what '1.2 miles' means."	Post images from the previous lesson of decimal numbers in different contexts. Post: "Drive east on route 66 for 1.2 miles."	
<u>Problem 1</u> Students will receive a map (#1) with a scale labeled at 100-mile intervals. Students will be directed to turn at 180 miles. "*Using your map and directions, determine at which street you would turn, and explain how you know.*"	Post enlarged copy of map. Have at least one student explain the solution.	Do all students share their thoughts as they examine the map?
<u>Problem 2</u> Students will receive a map (#2) with a scale labeled at 10-mile intervals. Students will be directed to turn at 25 miles. "*Using your map and directions, determine at which street you would turn, and explain how you know.*"	Post enlarged copy of map. Have at least one student explain the solution.	Do students recognize that 25 miles is equidistant between 20 and 30 miles and use that to determine which road to turn on?
2. Posing the Problem Students will receive a map (#3) with a scale labeled at 1-mile intervals. Students will be directed to turn at 1.2 miles. "*Using your map and directions, determine at which street you would turn, and explain how you know.*"	Post enlarged copy of map. Clarify that the numbers in this problem and on the map contain "dots" [decimal points]. Remind students that they encountered numbers like these during the previous lesson.	
3. Anticipated Student Responses S1: Estimate 1.2 less than halfway from 1 to 2 and choose Birch St. [correct] S2(a): Make 9 tick marks between 1 and 2 and choose Birch St. [correct] S2(b): Make 10 tick marks between 1 and 2 and choose Birch St. S3: Count two streets after 1 mi and chooses Cedar St. S4: Divide the interval into two parts; choose Cedar St. S5: Misread 1.2 as twelve. "It's not on the map." S6: (stuck)	S1: Push students to articulate how they know this is the correct thing to do. S2(b): "What numbers would you put next to these marks?" S3, S4: "What if the directions said 1.5 miles? Would that be possible?" S5: Point out the decimal point. "Remember that you saw numbers like this yesterday." S6: Like S5, remind the student of experiences in the previous lesson.	Do students make a connection to their experiences with decimals the previous day? Do they relate this map to the previous ones?

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
4. Comparing and Discussing If groups were stuck, invite them to explain why they had trouble with this problem. Order of presentation: S3, S4, S1, S2 (prefer (b) so as to correct this misconception). Have students add to their K-W-L chart ("Know, Want to know, Learned") from the previous lesson.	In the case of S2(b), with the class, clarify what numbers go with each tick mark so that students see that there is one mark too many. "So really we only need 9 tick marks." Teacher then makes a corrected version.	Are students able to identify the mistake and explain why it is incorrect?
5. Summing up On map #1, the teacher makes nine tick marks between 100 mi and 200 mi. With input from the class, she labels them 110, 120, etc. On map #2, make nine tick marks between 10 and 20 and label them 11, 12, etc. On map #3, make nine tick marks between 1 and 2 and label them 1.1, 1.2, etc.		Do students make a connection to their experiences with decimals the previous day?
Introduce the terms "decimal point," "decimal numbers," and "whole numbers." "*By using a decimal point, we can express numbers that are between the whole numbers, like the way we write numbers between 100 and 200 or between 10 and 20."*	Post vocabulary cards: decimal point, decimal numbers, whole numbers. Post: "By using a decimal point, we can express numbers that are between the whole numbers."	
6. Assessment Students will each receive map #4 (identical to map #3) and will be asked to identify the street at 2.2 miles. Students will also be asked, "*If you begin at the star, how far is it to Park St.?"*		Do students apply what they have learned so far? Can students justify their choice to their peers?

8. Evaluation

Was there evidence that students' experiences with decimals in the first lesson were valuable for understanding decimals in this lesson?

Is there any evidence that students see decimal numbers as a natural extension of the whole number system?

9. Blackboard Plan

What does "1.2 miles" mean?

Map #1	(student solutions)
Map #2	(student solutions)
Map #3	(student solutionns)

New terms

Decimal point

Decimal numbers

Whole numbers

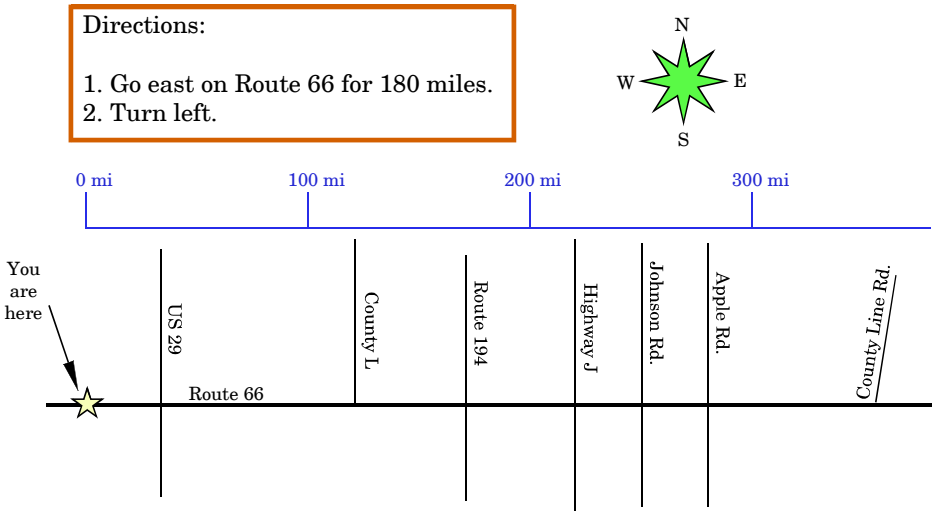
By using a decimal point, we can express numbers that are between the whole numbers.

Images from prev. lesson

Map #1

Directions:

1. Go east on Route 66 for 180 miles.
2. Turn left.



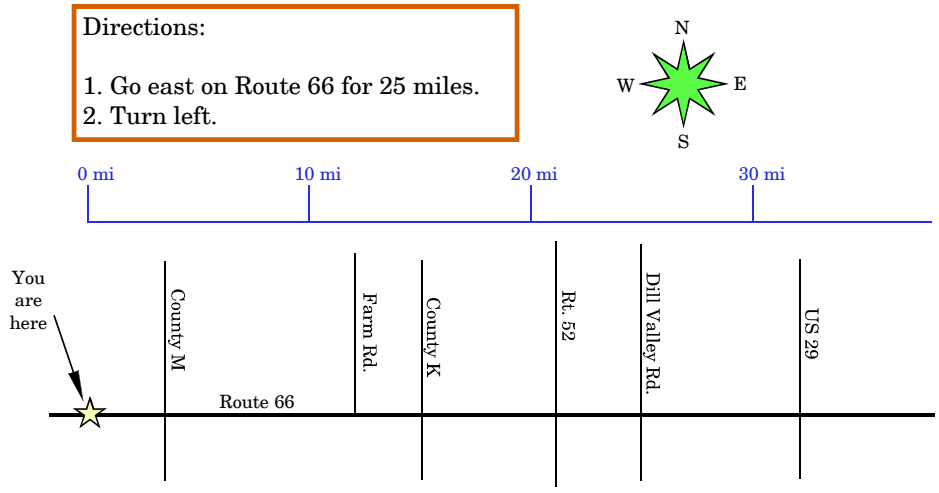
Based on the directions, what road should you turn on?

Explain how you know.

Map #2

Directions:

1. Go east on Route 66 for 25 miles.
2. Turn left.



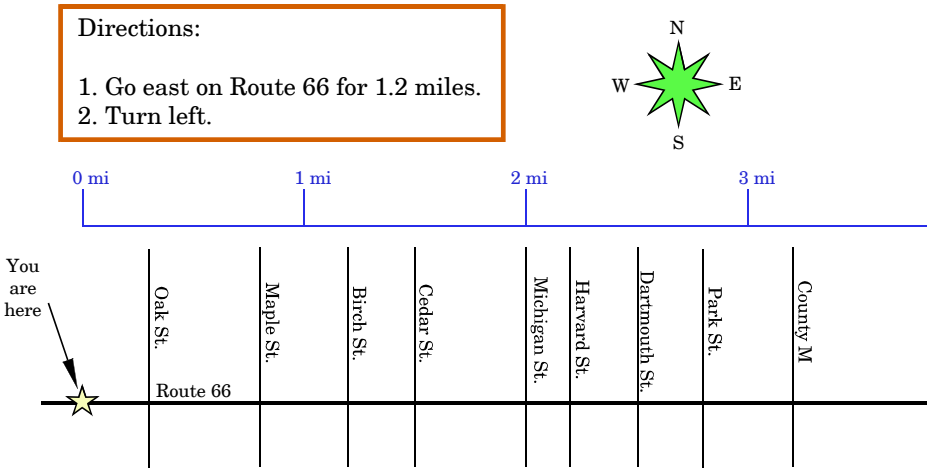
Based on the directions, what road should you turn on?

Explain how you know.

Map #3

Directions:

1. Go east on Route 66 for 1.2 miles.
2. Turn left.



Based on the directions, what road should you turn on?

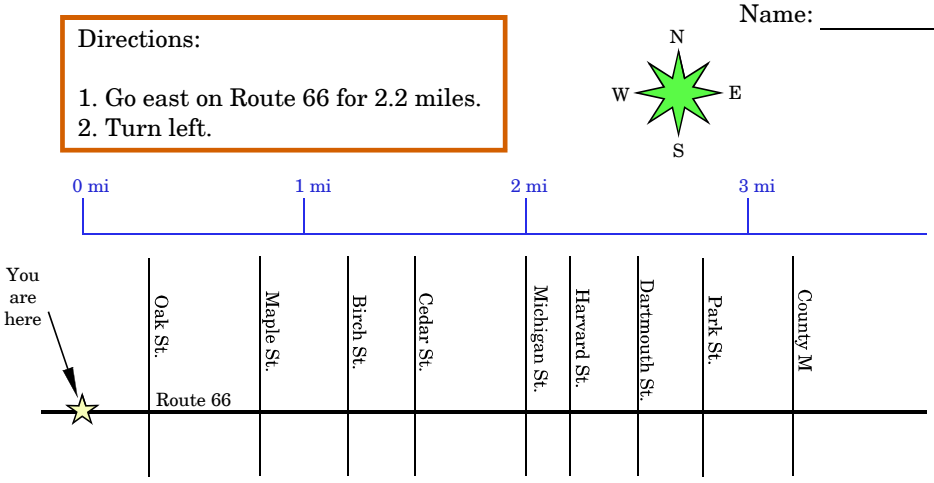
Explain how you know.

Map #4—Assessment

Name: _____

Directions:

1. Go east on Route 66 for 2.2 miles.
2. Turn left.



A. Based on the directions, what road should you turn on? _____

B. Explain how you know. _____

_____

C. If you begin at the star, how far is it to Park St.? _____ miles

Name:

Gas Pump

Watch the movie. Part of it will repeat, pausing twice for you to write some things down.

(First pause)
1. a. Write down what the gas pump shows:
Sale $ _____
Gallons _____

b. Do you think the amount of gas pumped so far is more than a gallon or less than a gallon? Explain why you think so.

_____

_____

_____

_____

(please continue on the next page)

Gas Pump, continued

(Second pause)
2. a. Write down what the gas pump shows:
Sale $ _____
Gallons _____

b. Now do you think that the amount of gas pumped so far is more than a gallon or less than a gallon? Explain why you think so.

_____

_____

_____

_____

3. Write down anything else you notice about how the numbers on the gas pump change.

_____

_____

_____

_____

Body Temperature

1. Ask the adult at this station:

What is a typical healthy body temperature? _____

2. Record your temperature: _____

3. List the temperatures of the other members of your group.

Name	Temperature
_____	_____
_____	_____
_____	_____

Stopwatch

Note: Don't use the green button. If you press it by accident, press it again until the word "LAP" disappears from the display.

1. Watch the second hand on the wall clock. Start the stop watch, then stop it 10 seconds later. The stop watch probably doesn't show just "10".

Repeat this three times, and write down exactly what the stop watch shows each time.

First time: _____

Second time: _____

Third time: _____

2. How long can you hold your breath? Ask another member in your group to time you. Write down exactly what the stop watch shows:

You can hold your breath for _____.

Write down what the stopwatch shows when your partners hold their breath.

Name	Time
_____	_____
_____	_____
_____	_____
_____	_____

Beans on a Scale

1. What is written on the large bag of white beans? _____

2. What is written on each large bag of black beans?_____

3. What is written on each small bag of black beans?_____

4. What does "lb" mean? (Ask an adult if you don't know.)_____

5. Put one large bag of black beans on the scale. Which of these matches what the scale shows? (Circle one)

10 1.0 01 0.1

6. Put one small bag of black beans on the scale. Which of these matches what the scale shows? (Circle one)

10 1.0 01 0.1

7. Weigh the bag of navy beans. Write down what the scale shows.

8. Use the bags of black beans to get the same weight as the navy beans. What bags did you use? How many?

Type of bag	How many
1 lb	
0.1 lb	

9. Is there another way to do it?

_____
_____

10. What did you learn?

_____
_____
_____
_____

Let's Look at these Numbers!

Getting Directions on the Internet

On Friday we are going to visit DePaul University. Let's find out how to get there!

1. On the computer, choose "Google Maps" from the "Favorites" menu (or go to **maps.google.com**).

2. Click "Get Directions".

3. For "Start address," type: 2201 N. Clark, Chicago
4. For "End address," type: 2300 N. Sheffield Ave.
5. Click "Get Directions."

6. Copy down the directions and number of miles!

1.		
2.		
3.		
4.		

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