6th grade public research lesson on May 22, 2004
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Mathematics Lesson for Grade Three

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Grade Sixth Mathematics Class

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1. Title of the Lesson: Patterns In Hexagon Tables

   Goal of the Lesson:

   To deepen students understanding about general patterns.
   To develop students algebraic habits of thinking.
   To encourage students to find calculating shortcuts in solving problems.
   To provide students an opportunity to develop equivalent expressions for a given problem.
   To help students become good problem solvers by providing a challenging open-ended problem

   The research lesson addresses the following Principles and Standards for School Mathematics (NCTM 2000) and Illinois Learning Standards for Mathematics.

Algebra Standard for Grades 6-8

Represent, analyze, and generate a variety of patterns with tables, graphs, words, and when possible, symbolic rules
Relate and compare different forms of representation for a relationship
Model and solve contextualized problems using various representations, such as graphs, tables and equations

Illinois Learning Standard – State Goal 8 – Grade 6

Use Algebraic and Analytical Methods to identify and describe patterns and relationships in data, solve problems, and predict results.

Chicago Academic Standard:

Extend, create, describe, and analyze geometric and number patterns
Describe trends, patterns, verbal rules, functions, and other mathematical relationships using tables, graphs, charts, and open sentences created from given or student-generated data

Instruction of the Lesson

NCTM Standards expect middle-grade students to learn algebra as the study of patterns relationships. Students in the middle grades should learn algebra both as a set of concepts and competencies tied to the representation of quantitative relationships and as a style of mathematical thinking for formalizing patterns, functions, and generalizations.

In this lesson students will be asked to observe a pattern and to generalize a relationship. Students will look for different ways to look at a pattern that produces different expressions. Students will be asked to construct a model and explain how it is put together, not in terms of numbers but in terms of its underlying physical structure. Students will be encouraged to develop justified generalizations of a pattern.

In early elementary grades my students' work has been focused on computation and developing number sense. Students still need opportunities to explore the properties of numbers and to explore context where objects are multiplicatively related to each other in order to understand the proportional reasoning skill often used in algebraic contexts. In this lesson, students will learn the specific procedures of arithmetic that are conceptually consistent with the generalized procedures of algebra.

Students in grade six can begin developing habits of mind that foster algebraic thinking by modeling quantitative situations so that the relations among variables become apparent. Students will model repeated patterns using hexagon tables to develop a general rule that models all situations of this type. Generalizing number patterns will help students create a strong connection between the Content Standard of Number and Operation and Algebra. Using the hexagon tables as an instructional model, students will build on the thinking of others and expand the domain of their inquiry through conversations with classmates.

Frequently students are given specific tables to look for patterns and formulate conjectures about what they observe. The hexagon models will give students an opportunity to formulate conjectures and problem solve. The lesson unfolds with a large group problem posing, then students' work in small groups to problem solve and finally there is a whole group discussion of students' findings and discoveries. The hexagon tables were chosen as a real-world model used in our classroom. Using the hexagon table model gives student an opportunity to find which factors vary and which remain constant which changing the number of tables that are arranged with the edges touching. This lesson will guide students toward an understanding of what constitutes a valid and useful algebraic generalization. The hexagon table model provides insight into the connections between the arithmetic and geometric relationships.

Unit Overview

Day 1: Order of Operations

Students review the rules for the order of operations by solving problems on a scientific calculator. They also evaluate expressions according to these rules.

Day 2: Review: Number Sentences

Students will review relation symbols, number sentences, and equations; to identify number sentences as true or false; and to translate between English and number sentences.

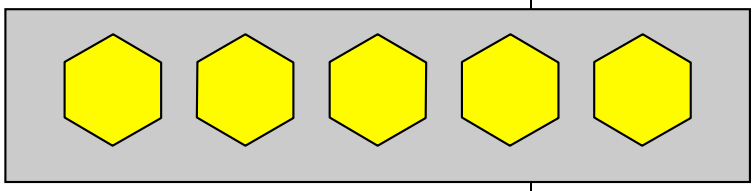
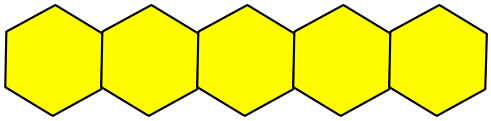
Day 3: Research Lesson

See Lesson Plan

Day 4: Using Variables to Describe Number Patterns

Students will describe a general number pattern in words and write examples, or special cases, of it. They are also given special cases for a general pattern and describe it with a number sentence having one variable.

Learning Process (or Lesson Plan)

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
1. Introduction Students are told the following: We are setting up for a family reunion in the park. We have 5 hexagon shaped tables. If one chair is on each side of each table, how many people can sit at the tables? 	Teacher displays five hexagons on the board. Ask for student response and teacher writes solutions on board. Anticipated solutions: 5x6=30 6+6+6+6+6=30 (6+6)+(6+6)+6=30 count all the sides Do students know what each number represents in their solutions?	Can students count number of sides and determine that there are 30 seats? Do students see the similarity in their solution methods? Do students agree on an easy strategy to find the number of seat?
Posing Problem Oh no! It started raining and we have to move the tables inside, but we have less room. We have to push the tables together to fit in the room. Now how many people can sit at these 5 tables?  Ask students to explain their solutions. Students will go to board to write the ways they found the number of seats.	Students will work in groups with hexagon shapes to solve problem. Teacher will walk among tables to see methods chosen. Encourage students to talk freely about their ideas for finding the number of seats with their partners. Students will record their thinking in their journal books. Teacher encourages students to develop different ways to solve problem.	Do students correctly put tables together using hexagon shapes? Do students recognize the loss of seats due to matching sides?
Solving Problem **Anticipated Students' Solutions** Count all sides Count all exposed sides 5+4+4+4+5=22 5+(3x4)+5=22 30-8=22 6x5=30 (5x4)+2=22	If students count all sides, teacher will guide students to look more closely at *available* seats. Teacher will encourage students to find different ways to solve problem. Make sure students know what each number represents in their solution.	Do students recognize that there are fewer seats available when tables are pushed together?

Extending the Problem Students will find the number of seats for 25 hexagon tables. Ask students to explain their solutions. Students are expected to use their previous knowledge as an easy way to find the number of seats for 25 tables. **Anticipated student solutions:** 5+(23x4)+5=102 (25x6)-(24x2)=102 (25x4)+2=102 Students will go to board to write their strategies used to solve the problem.	Teacher will guide students to use previous solutions to find the number of seats for 25 tables. Students will record their thinking in their Math journal books.	Do students use the solutions from the previous problem to find the number of sides for 25 tables? Are students able to write an equation to find the number of seats in 25 tables? Do students know what each number represents in solution?
Comparing and Discussing • Students will be given the time to work with partners to find the number of seats in the hexagon table arrangements • Ask students to explain how they found the number of seats in the hexagon table arrangements and record in journal books. • Students will be called to the board to explain their strategies.	• Ask students to explain their strategies and record their strategies on the classroom board in order to help other students understand the solution to the problem • The students called to the board will represent the different strategies students used. • Make sure students know what each number represents in their solution.	Are there any groups that did not find a easy strategy to find the number of seats? Do all the groups use the same strategy to find the number of seats? Do all the groups understand the different solutions to find the number of seats? Does the class agree on an easy method to find the number of seats? Can students construct a general rule for a given number of tables?
Summing up Using the writing on the board, review what students learned through the lesson. Ask students to write a journal entry about what they learned through this lesson.		

Evaluation

There are two important goals of the lesson. First, students must find the number of seats for tables of different lengths before asking them to construct a general rule. This progression helps students identify which factors vary and which remain the same when calculating the number of seats. Second, students were required to find the number of seats for a given number of tables. Extending the lesson by having students find hexagon tables of longer lengths, forces students to move beyond using the hexagons models and counting strategies towards identifying a general relationship that exists in the problem.

Generalizing numeric situations gives students an opportunity to engage in discussions about important mathematical ideas. While students did not use symbols, they did develop their own representational schemes to organize their thinking and build a sense of variable. The students were engaged with a problem that was posed in a context that made sense to them. Students were allowed to explore the problem on their own and to find the different ways to reason about the problem and the underlying algebraic thinking.