4th Grade Mathematics Lesson Plan

April 16-18, 2002
Brewer Island School, San Mateo, CA
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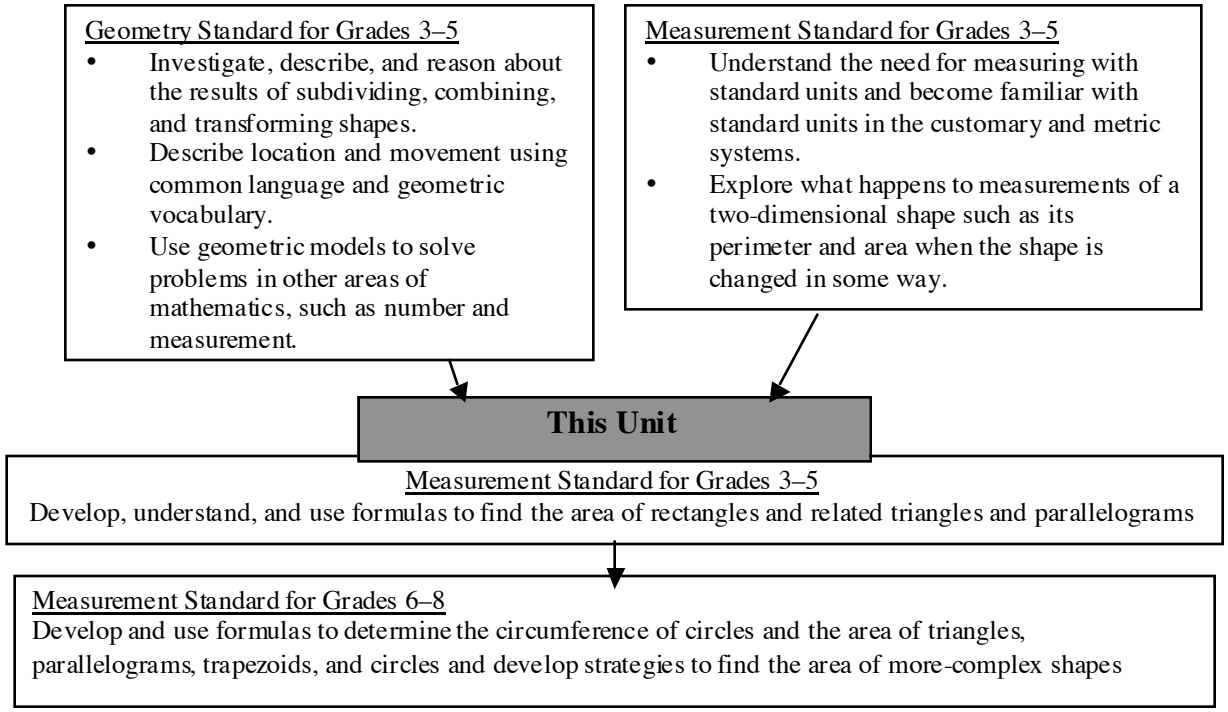
1. Title of Unit: Finding the Area of Shapes

2. Goal:
   a. To deepen students' understanding of the concept of area through problem solving activities.
   b. To develop the concept of equivalent-area transformation to use as the basis for finding the formulas for the area of a parallelogram, a triangle, and a trapezoid.
   c. To recognize that the area of a shape can be found by transforming it into a rectangle or a square.
   d. To help students become good problem solvers by providing a challenging open-ended problem.
      i. To encourage students to use existing knowledge to solve a challenging problem.
      ii. To encourage students to see common properties and relationships among various solutions presented by their peers in order to find a better solution to the problem.
      iii. To encourage students to look at their solutions from a different perspective and develop their ability to use logical reasoning to make conjectures by exposing them to their peer's different solutions.
   e. To provide opportunities for students to recognize the importance of working with their peers in order to deepen their understanding of mathematics.

3. Relationship between this Unit and the Principles and Standards for School Mathematics (NCTM 2000).



4. Instructional Plan

Shapes and Area: Three lessons total

i. How to find the area of a shape using previously learned knowledge (1)
ii. How to find the area of a shape using previously learned knowledge (2)
iii. Making shapes that contain an area of eight square units on the geoboard

5. Instruction of the Lessons

A series of lessons consisting of three periods of hands-on problem solving activities was planned to help students extend existing knowledge and develop a foundation for finding the formulas for the area of a parallelogram, a triangle, and a trapezoid.

These lessons were based on the idea of student-centered instruction which the NCTM Standards (2001) supports. This idea is expressed in the objective: "To provide students with an opportunity to develop a formula for the area of parallelogram meaningfully through investigation." According to the Standards, "Students can develop formulas for the area of a parallelogram and a triangle by using what they have learned previously." This prior knowledge includes the formula to find the area of a rectangle and the concepts of decomposing a shape and rearranging its components to make more familiar shapes without changing the area of the original shape. Although the Standards do not provide detailed information on how a teacher can help students to develop formulas for finding the area of parallelograms, triangles, and trapezoids meaningfully, it is important for students to have the opportunity to investigate and experience how those formulas can be developed by incorporating their previously learned knowledge.

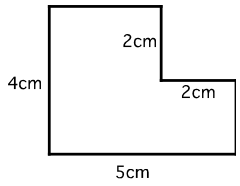
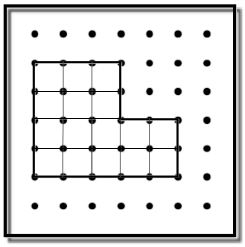
One way to help the students to develop these formulas through such investigation is to provide them with a series of problem solving lessons using manipulatives to develop the concept of equivalent-area transformation. Equivalent-area transformation is a key concept necessary to develop the formulas for finding the area of parallelograms, triangles, and trapezoids. This idea is developed by incorporating students' prior knowledge, such as the formula for finding the area of a rectangle and a square. Therefore, it might be a good idea to provide lessons designed to help students develop the concept of equivalent-area transformation prior to lessons designed to investigate the formula for finding the area of parallelograms, triangles, and trapezoids specifically. In order to do so a series of open-ended problem solving activities were developed based on Japanese problem-solving teaching methods/activities that are commonly used in Japan.

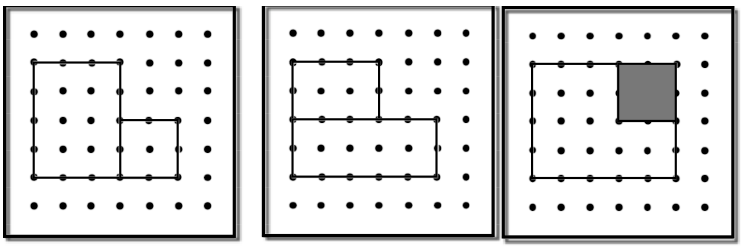
The Geoboard was chosen as the main manipulative for this series of problem-solving activities because of the following reasons:

- It is a manipulative that helps students to understand the relationship among various shapes and their areas.
- It has been one of the most commonly used manipulatives in mathematics education and various reform curricula have included it as a tool to provide students with opportunity to explore mathematical ideas.

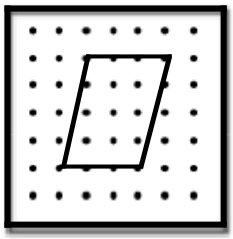
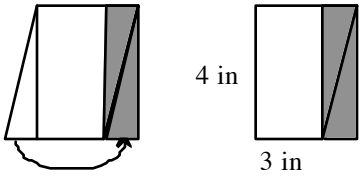
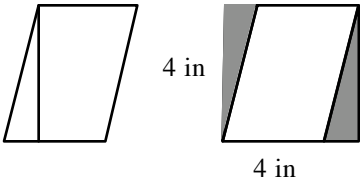
6. Lesson Procedure

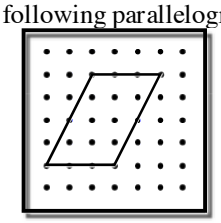
(1) How to find the area of a shape by using the knowledge that we have learned (1)

Learning Activities Teacher's Questions and Expected Students' Reactions	Teacher's Support	Points of Evaluation
1. Introduction Make this rectangle on your geoboard and find the area.  • Counting the number of unit squares on a geoboard • Using the formula for finding the area of rectangle, $L \sqcap W$.	Let students make the rectangle on a geoboard by assuming the distance between two pegs is one inch. Help students recall how to use a geoboard. Help students recall the formula for finding the area of a rectangle, $L \sqcap W$, and ask students to explain what this formula means.	Do the students recall how to use a geoboard? Do the students recall the formula for finding the area of a rectangle as well as the idea behind the formula?
2. Posing the Problem Make the following shape on a geoboard and find the area.  	Let students make the shape. Give each student a worksheet with a picture of the shape on a geoboard so that students can record their ideas about finding the area. Let students know that any ideas about finding the area based on their previous knowledge are acceptable.	Do students understand the problem?
3. Individual Problem Solving a. Determine how to find the area of the shape by: Counting the number of small squares in the shape. 	Encourage students to find a couple of different ways to find the areas. Provide students with worksheets to keep their work for whole class discussion. Encourage students to recall what they learned in previous mathematics lessons.	Can each student find at least one way to find the area?

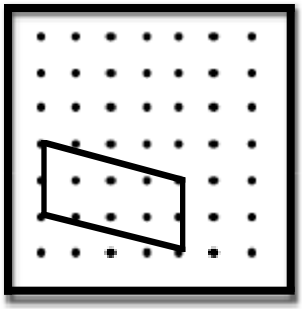
b. Dividing or transforming this shape into the shapes that students already know how to find the area by using a formula.  . 4x3 + 2x2=16 2x3 + 2x5=16 4x5 – 2x2=16		
3. **Comparing and Discussing Students' Solutions** (1) Ask students to explain their solution methods to the class. (2) Facilitate students' discussion about their solutions in order to understand their ideas behind each solution.	Write each student's solutions and on the blackboard in order to help students understand the discussion.	Can each student understand that there are several ways to find the area?
4. **Find a strategy to solve this kind of problem** The area of the shape can be found by dividing the shape into rectangles and squares, or transforming it into a rectangle or a square.		
5. **Summing up and Journal Writing** (1) Using the writing on the blackboard, review what students learned through the lesson. (2) Ask Students to write a journal entry about what they learned through this lesson.		

(2) How can we find the area of a parallelogram by using the knowledge that we have learned?

Learning Activities Teacher's Questions and Expected Students' Reactions	Teacher's Support	Points of Evaluation
1. Introduction to the Problem • Review the previous lesson. Ask some students to read their writing from the previous lesson. • Make the following shape on a geoboard 	By using selected student journal entries, which all the students wrote at the end of the previous class, help students to recall what they have learned. Provide each group of students with a picture of the shape on a geoboard, and let students make the shape on their geoboards by looking at the picture.	Can students make the shape on their geoboard?
2. Posing the Problem • Find the area of the shape on the geoboard by assuming that the distance between two pegs is one inch.		Do students understand the problem?
3. Individual Problem Solving Find the area of the parallelogram on a geoboard by: a. Dividing the shape into a triangle and the other part of the parallelogram and then cutting the triangle from the parallelogram and moving it to the other side of the shape to transform a parallelogram into a rectangle without changing its area.  **b.** Calculating the rectangle that covers the parallelogram and subtracting the area of the two triangles.  	Encourage students to use the knowledge that they learned previously to find the areas. Provide students with worksheets that have a picture of the shape on a geoboard so they can record their work for the whole class discussion. Prepare enough worksheets so students are able to have as many worksheets as they want.	Can each pair of students find a way to find the areas?
4. Discussing Students' Solutions By examining several student's solution methods, students are expected learn that a key idea in finding the area of the parallelogram based on their previous knowledge is equivalent-area transformation. a. Ask students to explain their solution methods to the other students in the class. b. Facilitate student discussion about their solutions, then lead students to realize that a key idea in finding the area of the parallelogram is equivalent-area transformation.	Write each student's solutions and ideas on the blackboard in order to help students understand the discussion.	Can students visualize the concept?

5. Exercises Find the area of the following parallelogram on a geoboard. 	Ask students if their solution methods can be used to find the area of another parallelogram on a geoboard. Help students understand that the area of this parallelogram is the same as the previous parallelogram.	
6. Summing up (1) Using the writing on the blackboard, review what students learned through the lesson. (2) Ask students to write a journal entry about what they learned through this lesson.		

(3) Making shapes with an area of eight square units on the geoboard

Learning Activities Teacher's Questions and Expected Students' Reactions	Teacher's Support	Points of Evaluation
1. Introduction to the Problem • Review the previous lesson. Ask some students to read their writing from the pervious lesson. • Make the following shape on a geoboard  • Find the area of the shape on the geoboard. 8 square inches	By using selected student journal entries, which all the students wrote at the end of the previous class, help students to recall what they have learned. Provide each group of students with a picture of the shape on a geoboard and let students make the shape on their geoboards by looking at the picture. Help students use ideas that they learned previously such as equivalent-area transformation.	Can students make the shape on their geoboard? Do students recognize that the shape on the geoboard is the same kind of shape that they worked on previously?
2. Posing the Problem Make a four-sided shape with the same area as this shape	To help all the students to understand this problem, give them several minutes to come up with some solutions. Then, examine some of those solutions as a whole class to see if any four-sided shape with an area of eight square units can be a solution. Therefore, there might be many different solutions.	Do students understand the problem?
2 Individual Problem Solving Anticipated student solutions: (See Appendix)	Provide each group of students several worksheets with a picture of the geoboard. Let students keep all the shapes that they make on their geoboards so that they can easily show them to the class during the whole class discussion.	Can students find at least one solution to the problem?

3. Discussing Students' Solutions (1) Ask students to explain why they think the shape has an area of eight square units. (2) After the students bring various shapes as their solutions, sort the shapes by using their previous knowledge. Their previous knowledge includes: Square: a four-sided figure with four right angles and four equal sides Rectangle: a four-sided figure with four right angles Parallelogram: a four-sided figure with two pairs of parallel sides Trapezoid: a four-sided figure with a pair of parallel sides Rhombus: a four-sided figure with four equal sides and two pairs of parallel sides	Ask students to bring and put their worksheets on the blackboard in order to help the other students understand what they have made and why the shape can be the solution to the problem. Help students realize that many four-sided shapes with an area of eight square units can be made on the geoboard. These shapes include not only a rectangle and parallelograms but also a square, trapezoids, and rectangles with different orientations.	Do students realize that several different quadrilaterals with an area of eight square units can be made on the geoboard?
5. Summing up (1) Using the writing on the blackboard, review what students learned through the lesson. (2) Ask Students to write a journal entry about what they learned through this lesson.		

Appendix: Anticipated Students Solutions