Curriculum Exploration (6b) Double Number Line Diagram

(Grades 3 – 6)

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Double Number Line in CCSS

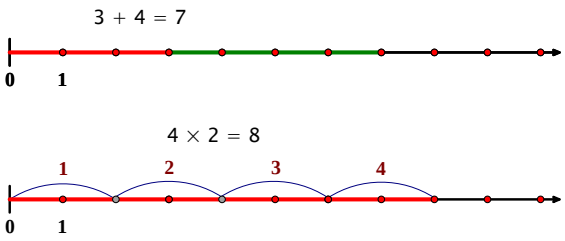
6.RR.3

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Why double number line in 3-5?

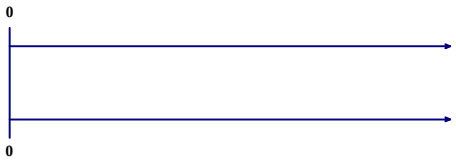
- If we want students to be able to use double number line to solve problems involving complex mathematical ideas like ratios and proportions, then students must be familiar with the tool before they reach Grade 6.
- Double number line diagram is a useful tool even for making sense of elementary school mathematics, particularly multiplication and division with fractions and decimal numbers.

Multiplication is two-dimensional



What is a double number line?

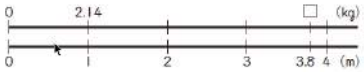
- A double number line is a visual representation of multiplicative relationship between two quantities.



Example

- Math International Gr. 5, p. A34

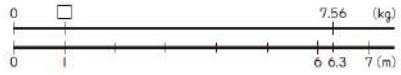
2 There is a 1 m pipe that weighs 2.14 kg. How much will 3.8 m of this pipe weigh?



Another Example

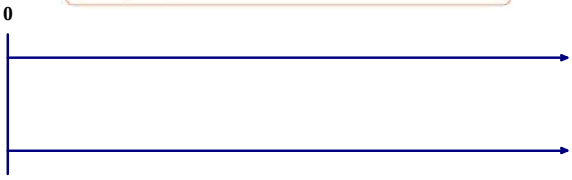
- Math International Gr. 5, p. A48

2 There is a 6.3 m iron pipe that weighs 7.56 kg. How much will 1 m of this iron pipe weigh?



Practice Problem

1 With 1 dL of paint, we can paint $\frac{4}{5}$ m² of boards. How many m² of boards can we paint with $\frac{2}{3}$ dL of this paint?



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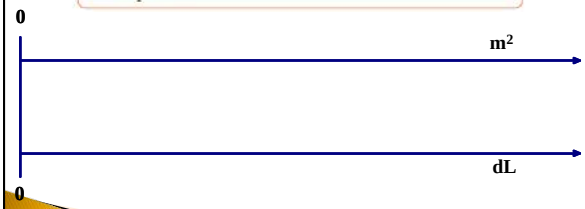

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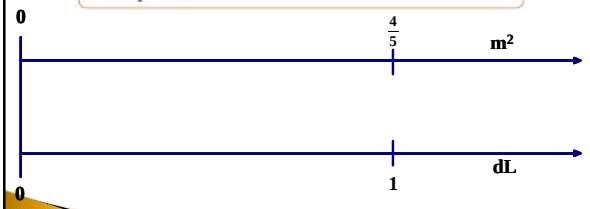

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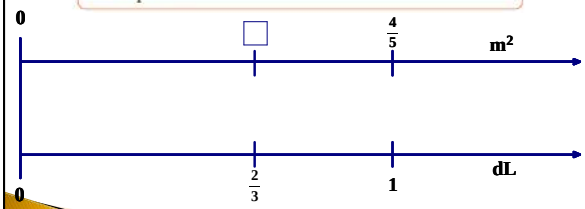

Two Types of Visual Model

- Models that represent the details of mathematical procedures.
- Models that represent the relationships of quantities → reasoning tools to decide what to do to solve problems.

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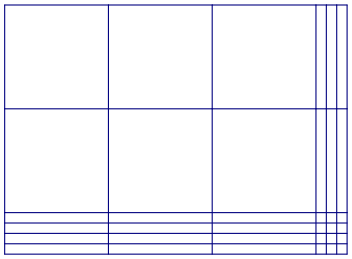
33×24



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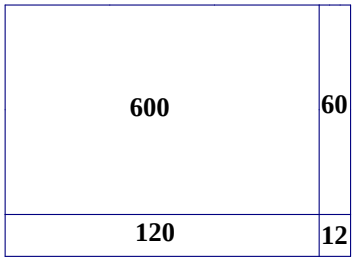
$$\begin{array}{r} 33 \\ \times\ 24 \\ \hline 132 \\ 66 \\ \hline 792 \end{array}$$



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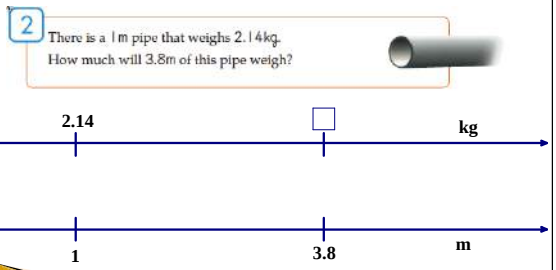

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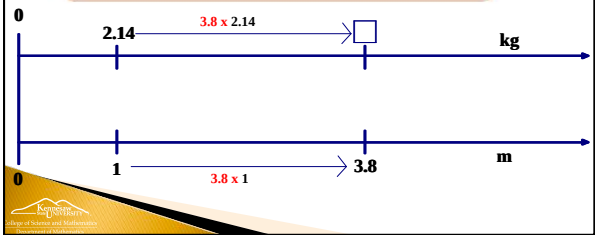

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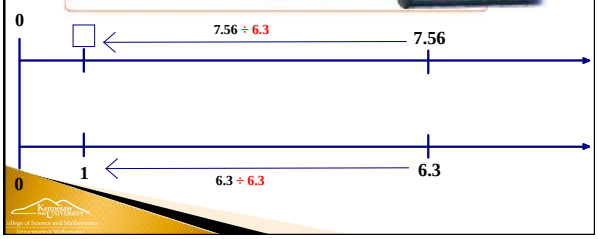

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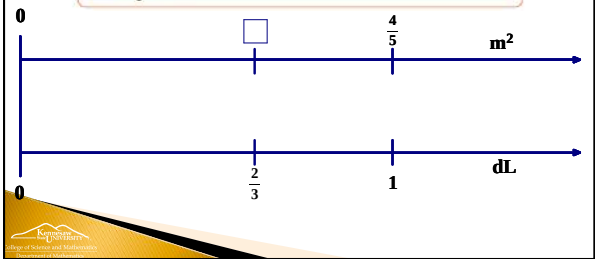

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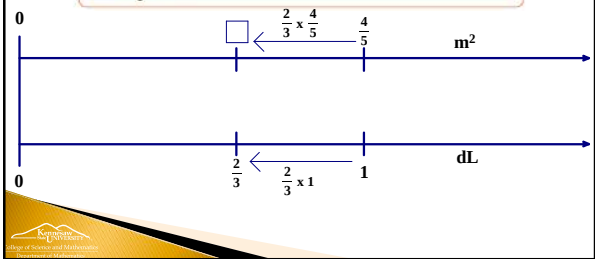
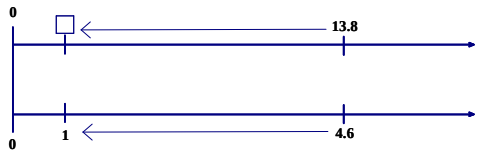


Key Properties of ×/÷

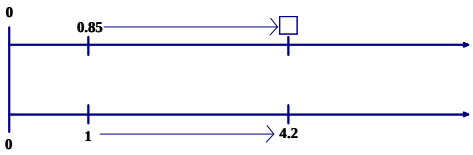
- $n \times 1 = n$: 1 multiplied by any number is the number itself.
- $n \div n = 1$: Any number divided by itself is 1.

What calculation is needed?



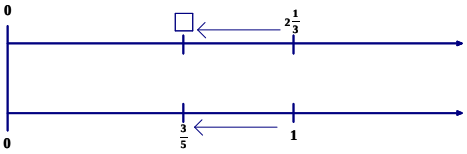
$1 = 4.6 \div 4.6$
→ $? = 13.8 \div 4.6$

What calculation is needed?



$4.2 = 4.2 \times 1$
→ $? = 4.2 \times 0.85$

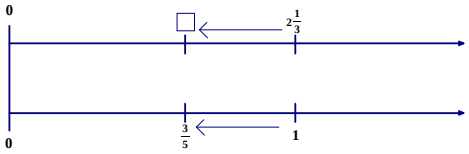
What calculation is needed?



$\frac{3}{5} = \frac{3}{5} \times 1$
→ $? = \frac{3}{5} \times 2\frac{1}{3}$

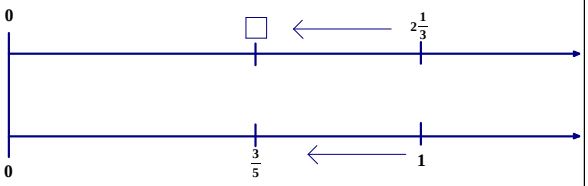
Common Misconceptions

- Multiplication makes bigger and division makes smaller.
- Students can tell that the unknown (answer) should be less than 2⅓, thus they believe division is the operation needed.



Reason about "how to calculate"

- ⅗ × 2⅓ = ?



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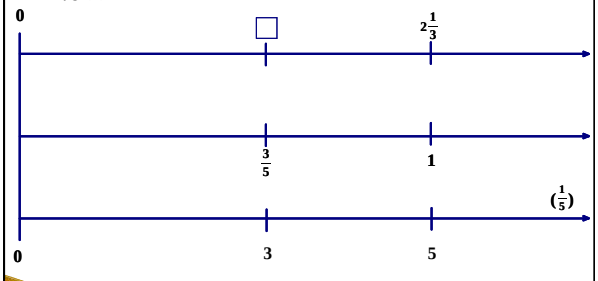

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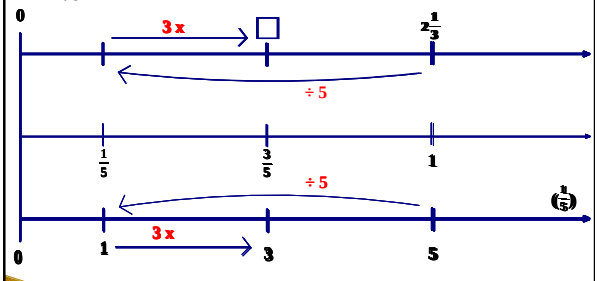
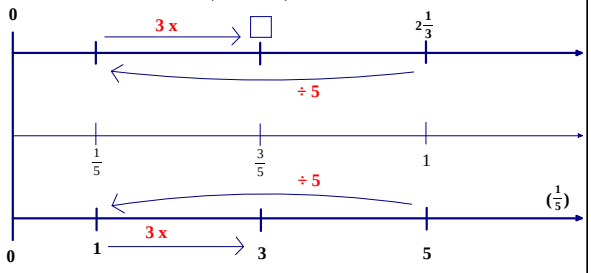


Reason about "how to calculate"

- ⅗ × 2⅓ = 3 × (2⅓ ÷ 5)



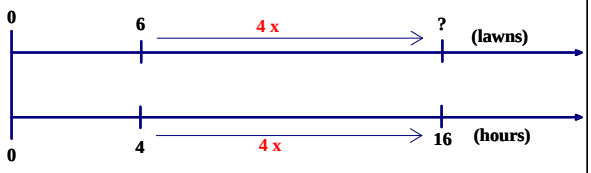
Reason about "how to calculate"

- ⅗ × 2⅓ = 3 × (2⅓ ÷ 5)
- CCSS 5.NF.4.a

  Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.

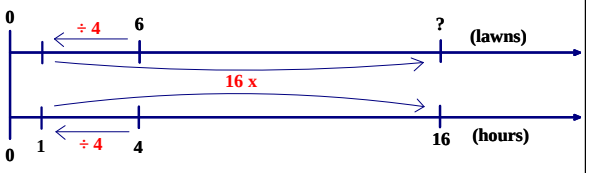
6.RP

- *If it took 4 hours to mow 6 lawns, then at that rate, how many lawns could be mowed in 16 hours?*



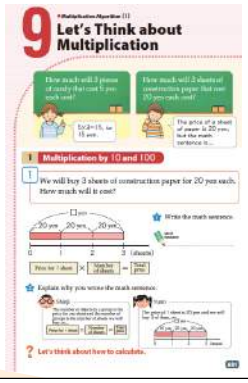
6.RP

- *If it took 4 hours to mow 6 lawns, then at that rate, how many lawns could be mowed in 16 hours? At what rate were lawns being mowed?*

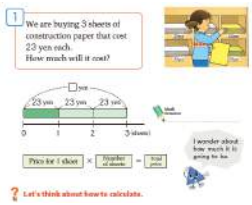


Developing Double Number Line

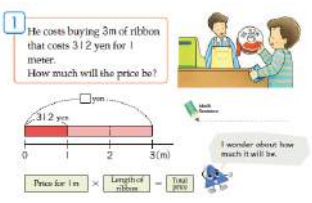
- Starting with a combination of a tape and a number line (Gr. 3 p. A91)



Representing Problem Situations

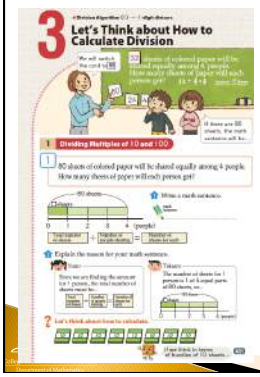


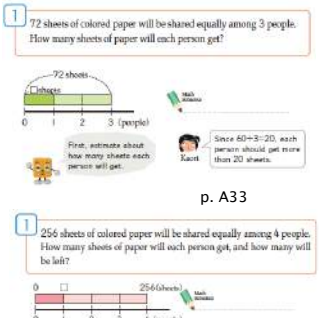
Gr. 3 p. A93



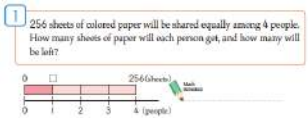
Gr. 3 p. A99

Grade 4 Division (Whole Numbers)



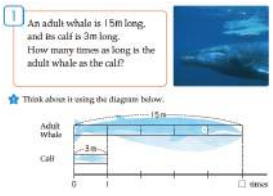


p. A33

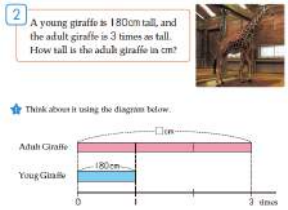


p. A41

× and ÷ as Scaling

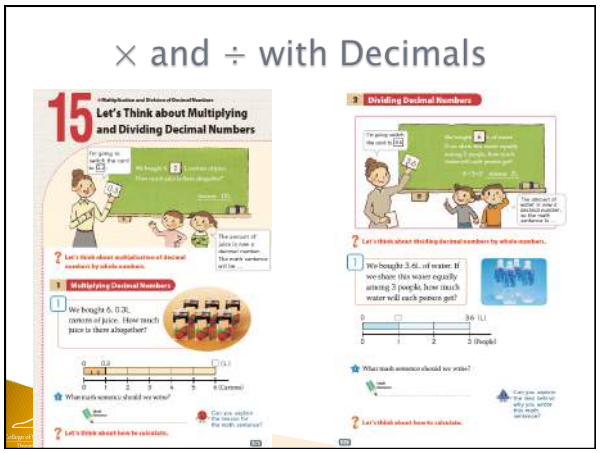


Gr. 4 p. A44

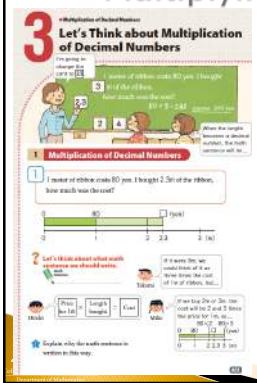


Gr. 4 p. A45

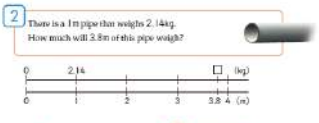
× and ÷ with Decimals



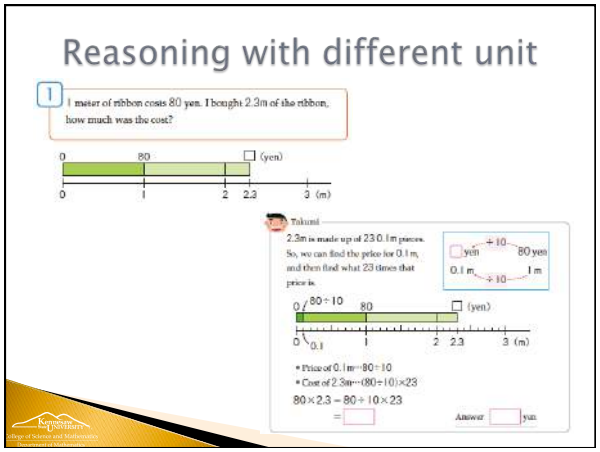
Multiplying by Decimals



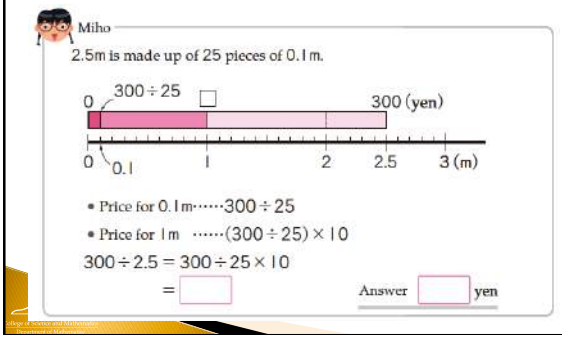
Double Number Line (Gr. 5 p. A34)



Reasoning with different unit



Reasoning with different unit Dividing by Decimals.



Big Ideas

- Visual representations such as double number lines are (can be) powerful reasoning tools.
- Developing a reasoning tool requires time.
- Collaboration across grades is essential.

Thank you.

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