Cultivating Mathematical Practices in a Community of All Mathematics Teachers

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Cultivating Mathematical Practices

- How do we cultivate a vision of mathematics teaching in which mathematical processes, proficiencies, and habits of mind play a central role?
- How do we develop and grow communities that support and nurture us to work towards Level 3 mathematics teaching (Takahashi, 2011)?
- What are key components of such a *professional mathematics teaching community*?

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Standards for Mathematical Practice in the Common Core State Standards

- MP1 Make sense of problems and persevere in solving them.
- MP2 Reason abstractly and quantitatively.
- MP3 Construct viable arguments and critique the reasoning of others.
- MP4 Model with mathematics.
- MP5 Use appropriate tools strategically.
- MP6 Attend to precision.
- MP7 Look for and make use of structure.
- MP8 Look for and express regularity in repeated reasoning.

An example: The concept of area

To cultivate mathematical practices we must consider:

- What is area? What does it tell us about a shape? What problems does area help us solve?
- What strategies can we use to calculate area? Why can we calculate area the way we do? Where do the common area formulas come from? What tools could we use to help us determine an area?
- How does area connect to other mathematical topics?

To cultivate mathematical practices, we must think *more deeply* about the mathematical ideas.

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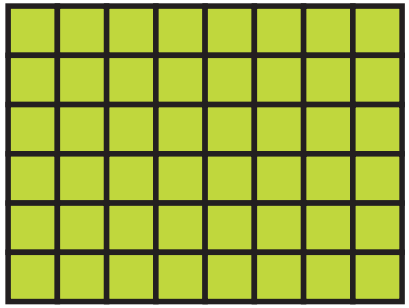
Area of rectangles

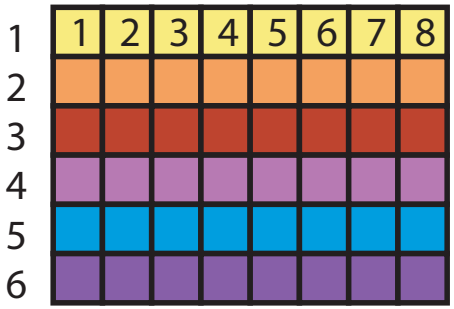
1 square unit

What is the area of this rectangle in square units?

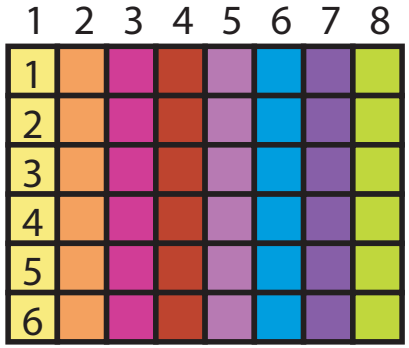
Cover the rectangle with squares. How many squares?



Is there a quicker way to find the area than counting all the squares one by one?



View as 6 groups of 8 squares.
$6 \times 8 = 48$ square units



View as 8 groups of 6 squares.
$8 \times 6 = 48$ square units

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Cultivating Mathematical Practices

The “Standards Movement” is rooted in a vision of deeper engagement in mathematical thinking:

- Mathematical Process Standards (NCTM)
- Mathematical Proficiency (NRC)
- Mathematical Practices (CCSSM)

We can view the Standards Movement as a yearning for a vibrant mathematics teaching community that will cultivate these practices.

Cultivating Mathematical Practices

A metaphor:

Lesson Study communities – seeds and shoots

Could a bigger community of all teachers of mathematics, from PreK through the college level be cultivated to support and grow these seeds and shoots?

Cultivating Mathematical Practices requires community

Habits of mind can develop in a community in which the habits are the norm.

I will argue that:
We need a community of mathematics teachers, from Prekindergarten through the college level, who discuss their work in depth, build on each other's ideas, and work towards a shared view of excellent mathematics teaching.

But why don't we already have this community?
And how do we develop this community?

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How do we develop the community that cultivates mathematical practices?

I will argue that:
Opportunities to gain peer recognition and high standing within the community *can* drive excellence.

External measures of assessment or oversight *don't* drive excellence.

I will explain how research in motivation informs us on these points.

Why a community of *all* math teachers?

The time is right for *all of us who teach math* to join together to become community:

- Math teaching is connected across all levels;
- we have the Common Core in common;
  the Common Core gives us a chance to “shake things up;”
- *all of us* have much to learn from each other about both math and math teaching;
- vibrant communities exist (Lesson Study!), let’s join them together for greater power.

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We have a common problem in math teaching

- The current system is not fostering excellence in math teaching *at any level*;
- math teaching is not regarded as a top-notch profession *at any level*;
- math teaching is not vigorous and vibrant the way some other mathematical professions are, such as math research.
- information that could lead to improvements doesn't seem to "take hold"
  - knowledge about mathematics,
  - knowledge about teaching and learning.

Why is it like this?

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Practices must grow from *within* a community

Example of how “external ideas” do not take root:

- Well-established research from decades ago that should inform math teaching is not as widely used as one would hope.

Examples of how “internal ideas” spur productive activity:

- In math research, important new theorems are immediately studied carefully and used in new results.
- In Japan, Lesson Study has been instrumental in producing a common vision of good teaching.

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Practices must grow from *within* a community

"Health systems worldwide are increasingly aware of the need to modernise service delivery and medical practice. While innovation is one road to modernisation, there also already exist mountains of clinical evidence and many examples of managerial improvements that have not yet become standard practice."

"Viewing the process as one of 'spread' implies a pushing out of ideas; a doing unto, by someone who knows more, to someone else. This model engenders resistance that slows change. This is largely due to the difference in view between those pushing out the change, the spreaders, and those receiving the demands, the potential adopters." (Fraser and Plsek, 2003)

Research on motivation

Internal motivation is fostered by environments that fulfill the basic human needs for

- autonomy
- competence
- relatedness

as opposed to environments that are externally controlling.

External measures of accountability cannot work in the long term because they will sap motivation. External reward structures are not as motivating as peer approval.

Research on motivation

"The picture that emerged from these meta-analyses of 128 well-controlled experiments exploring the effects of extrinsic rewards on intrinsic motivation is clear and consistent. In general, tangible rewards had a significant negative effect on intrinsic motivation for interesting tasks, and this effect showed up with participants ranging from preschool to college, with interesting activities ranging from word games to construction puzzles, and with various rewards ranging from dollar bills to marshmallows."
(Deci, Koestner, and Ryan, 1999)

Research on motivation

"...contingent, tangible rewards and other extrinsic factors such as competition and evaluations can be detrimental to outcomes such as creativity, cognitive flexibility, and problem solving which have been found to be associated with intrinsic motivation ..."
(Gagne and Deci, 2005)

Research on motivation

"...work climates that promote satisfaction of [autonomy, competence, relatedness] will ...yield the important work outcomes of (1) persistence and maintained behavior change; (2) effective performance, particularly on tasks requiring creativity, cognitive flexibility, and conceptual understanding; (3) job satisfaction; (4) positive work-related attitudes; (5) organizational citizenship behaviors; and (6) psychological adjustment and well-being."
(Gagne and Deci, 2005)

Research on motivation

"...pressures from schools, communities, and society for teachers to be more accountable for students' achievement can lead teachers to be more controlling and thus can be counterproductive for the goals of conceptual understanding and personal growth."
(Deci, Vallerand, Pelletier, and Ryan, 1991)

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How can math teaching become a vigorous, vibrant profession that cultivates mathematical practices?

What makes math research so strong and productive?

- **Public, shared knowledge**
  Math researchers publish their work, they discuss their work in depth, they build on each other's work;
  all can participate in publishing;
- **A reward structure of *peer approval***
  Quality is judged from *within* the community by peer recognition and admiration;
  the community is a meritocracy that is led by and learns from the top professionals in the field.
- **A high bar to entry**
  Entry into the community requires a high level of education and accomplishment.

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How could a community of all math teachers work together towards cultivating mathematical practices?

Let's set a high bar for entry into our profession:

Before entering the profession, teachers should study the mathematics they will teach in depth, from a teacher's perspective.
(See *The Mathematical Education of Teachers II* at cbmsweb.org.)

This is a *necessary* condition for high quality mathematics teaching, although it is not sufficient.

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How could a community of all math teachers work together towards cultivating mathematical practices?

All of us who teach math:

- Let's develop repositories of shared, vetted knowledge about math and its teaching;
- let's compete for each other's admiration through the sharing of ideas;
- let's develop a system that allows knowledge about math and its teaching to build and evolve;

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What kind of shared, vetted knowledge might we produce together?

- Research lessons from Lesson Study;
- Sample tasks for students;
- Short essays (like a blog post) that are revised in response to comments from the community;
- Video clips (or transcripts) of teaching and accompanying discussion/commentary;
- Commentary on existing resources;
- Other ideas?

Importantly:
These must be vetted by the community and they must be publicly available.
We must develop a culture *among all of us who teach math* of sharing knowledge about math teaching and building on the knowledge of others.

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Example: connecting area with algebraic expressions

Some opportunities to engage in mathematical practices in the CCSS:

- 5.OA: Write and interpret numerical expressions.
- 5.OA.2: Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.
- MP2: Reason abstractly and quantitatively.
- MP7: Look for and make use of structure.

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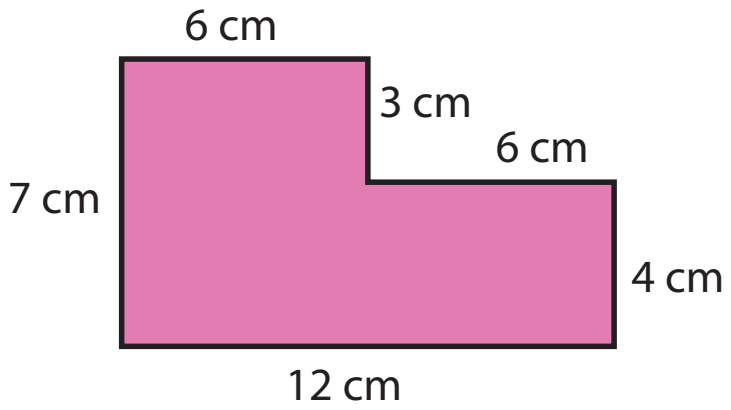
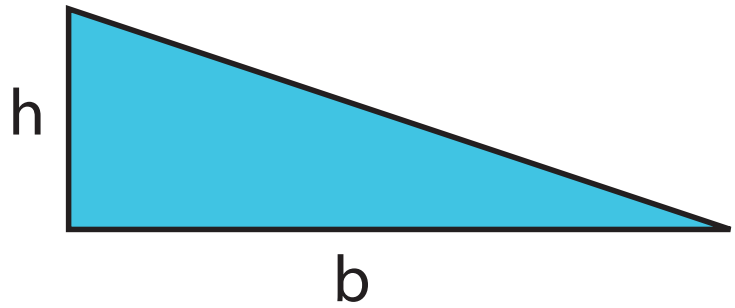
Example: connecting area with algebraic expressions

What is the area of the shaded shape?

Write expressions that correspond with different ways of finding the area.





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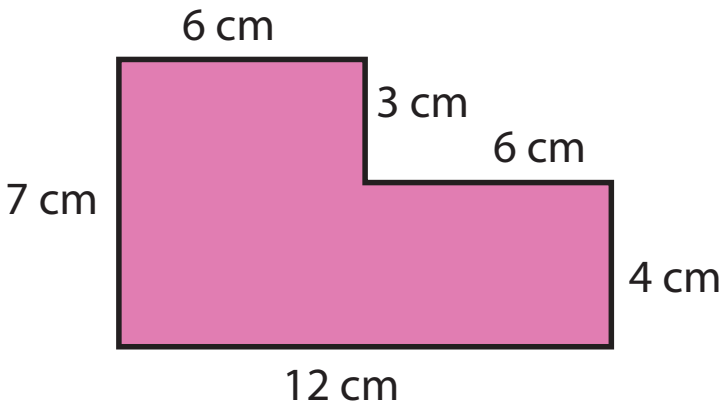
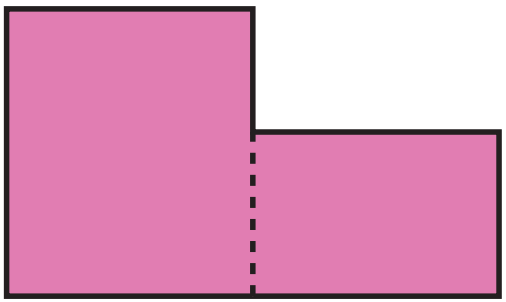
Areas of regions composed of several rectangles

What is the area of the shaded shape?

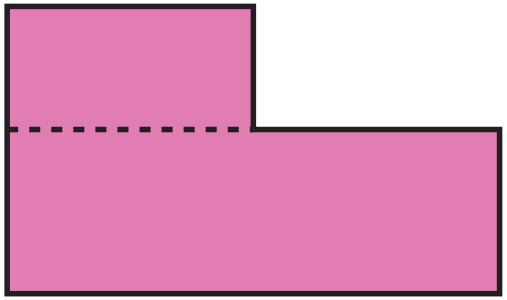


Method 1



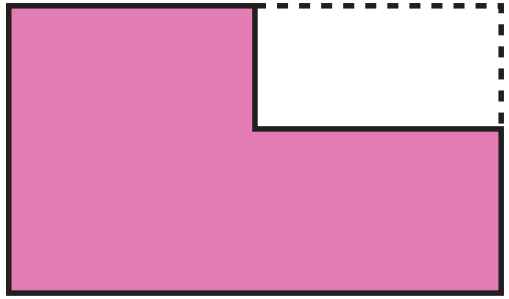
$7 \times 6 + 4 \times 6$

Method 2



$3 \times 6 + 4 \times 12$

Method 3



$7 \times 12 - 3 \times 6$

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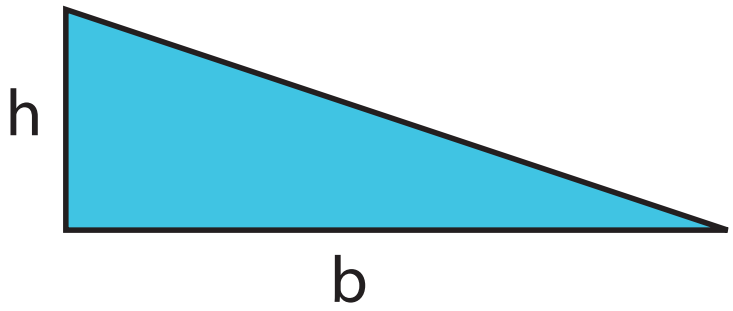
The area formula for triangles

One method:



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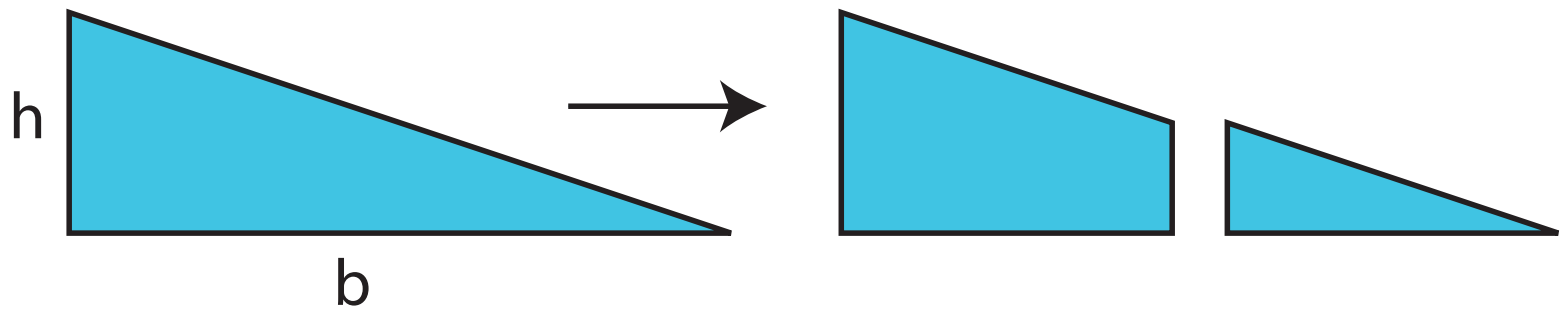

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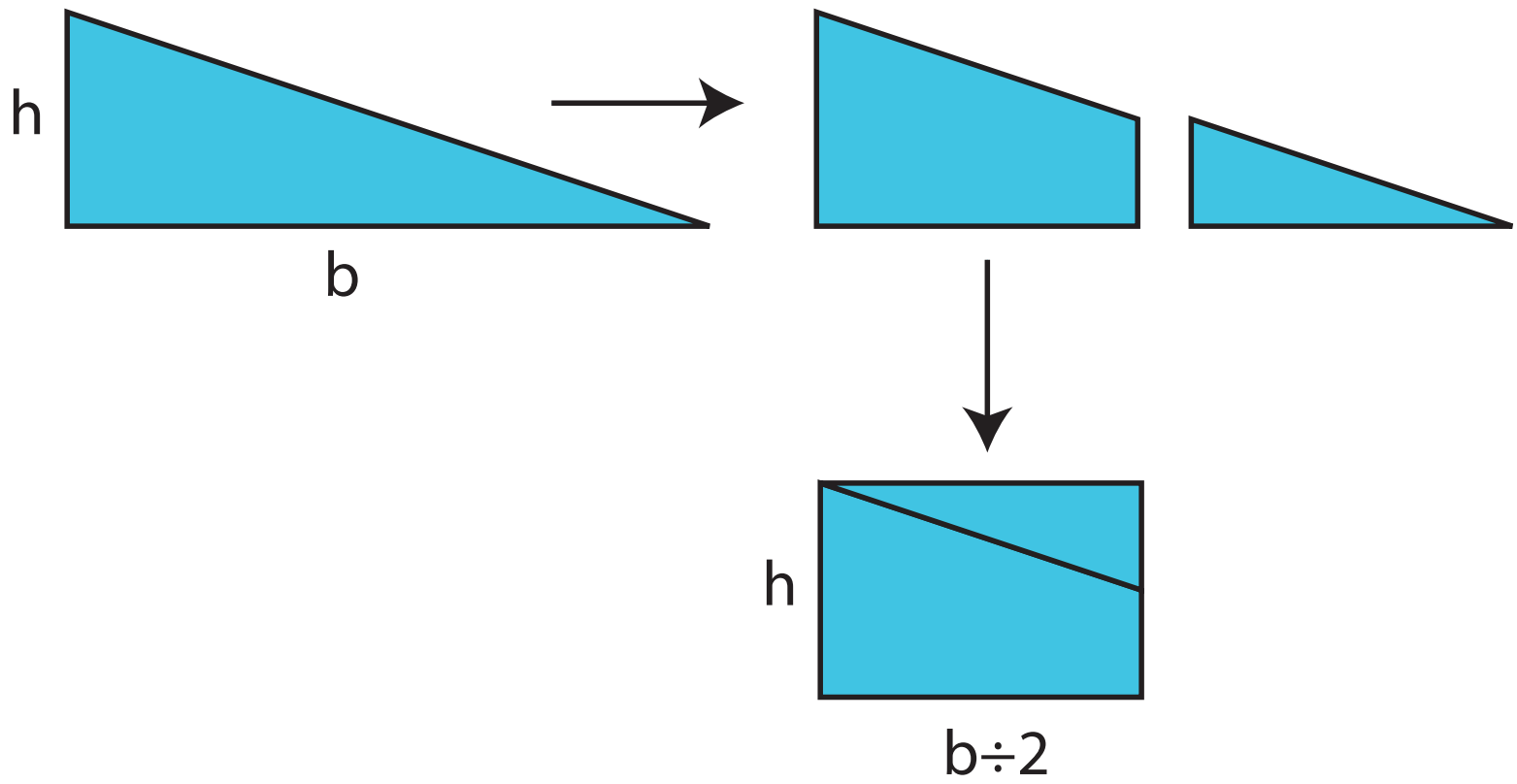
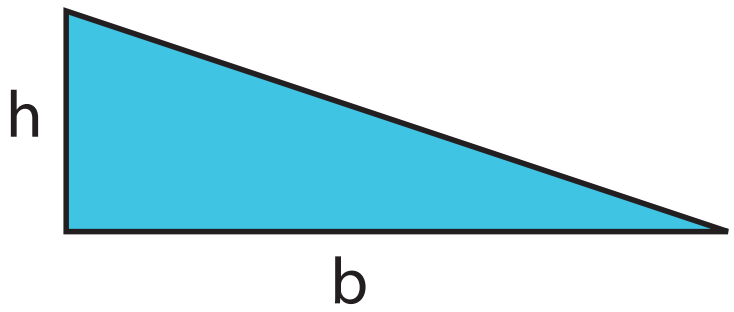


The area formula for triangles

Another method:



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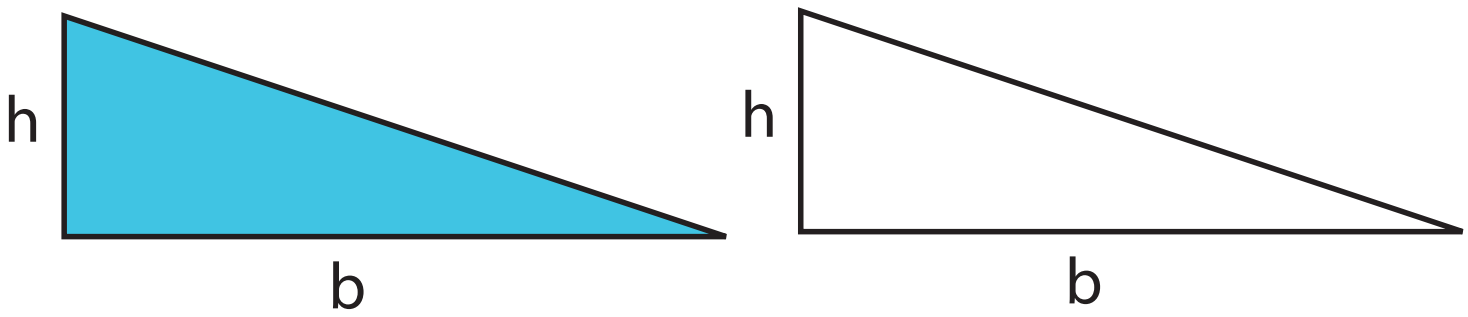

The area formula for triangles

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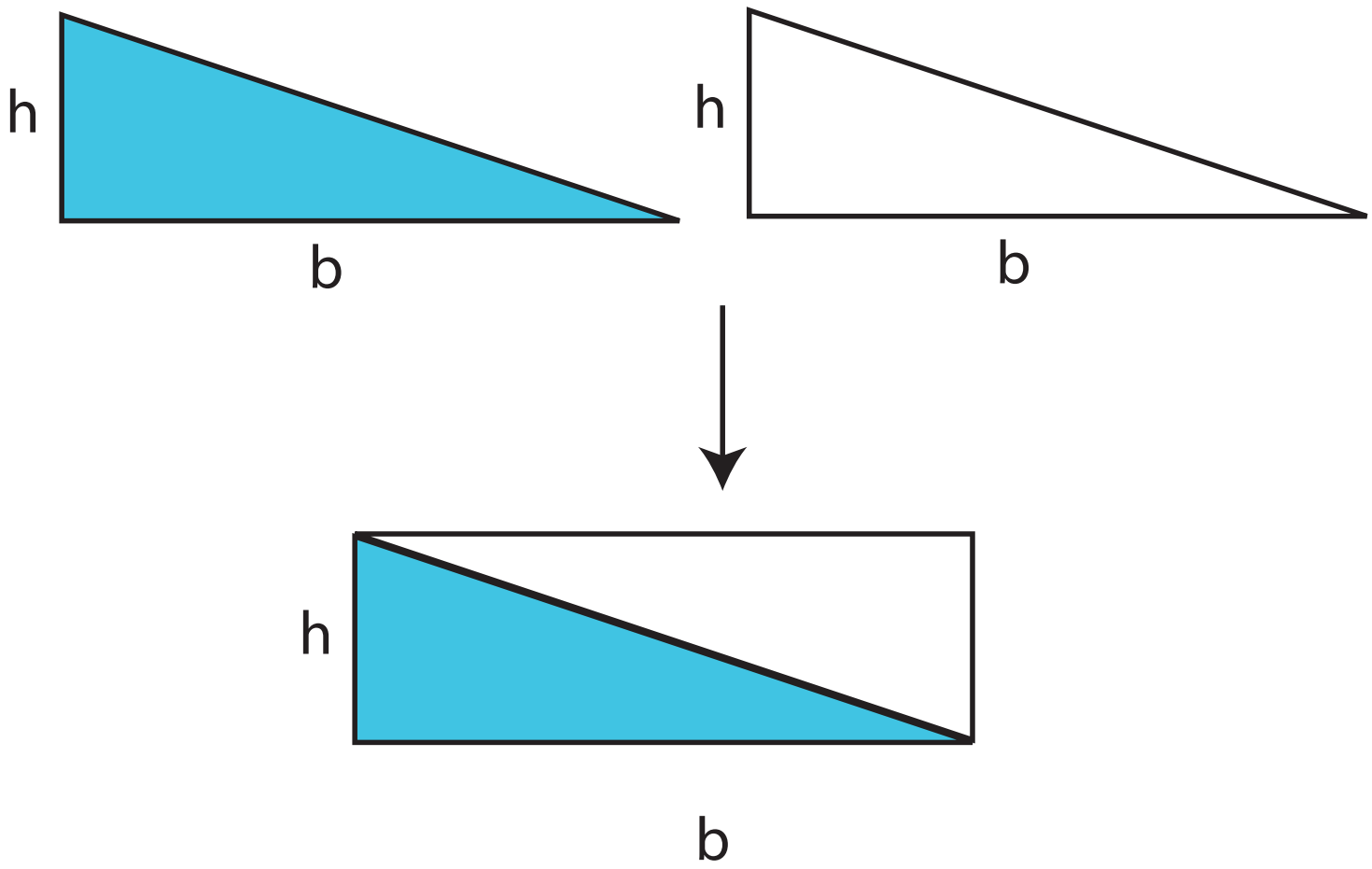

Connections between reasoning in geometry and in arithmetic/algebra

Expression for the area of the triangle from the first method:

$$(\frac{1}{2} \cdot b) \cdot h$$

Expression for the area of the triangle from the second method:

$$\frac{1}{2} \cdot (b \cdot h)$$

$$\updownarrow$$

$$5 \cdot 86 = (\frac{1}{2} \cdot 10) \cdot 86 = \frac{1}{2}(10 \cdot 86)$$

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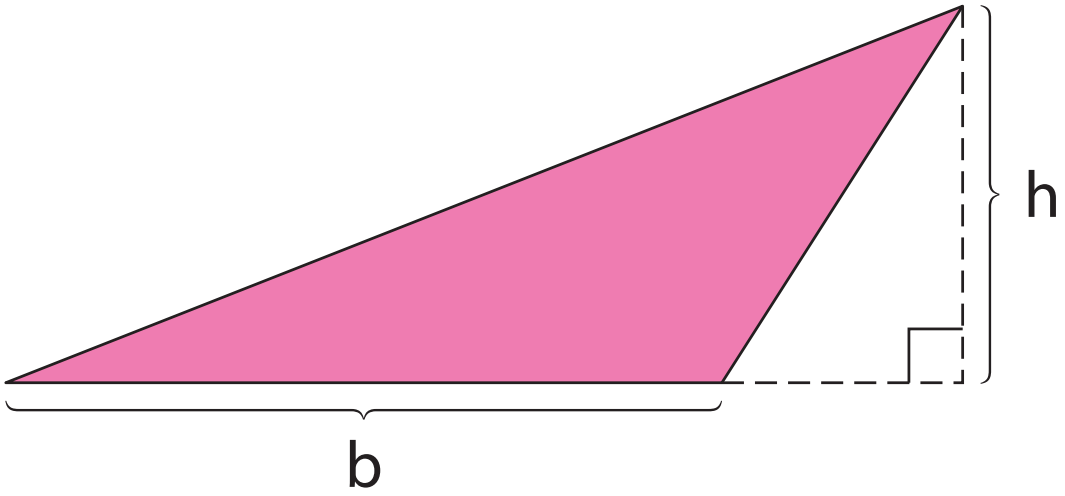
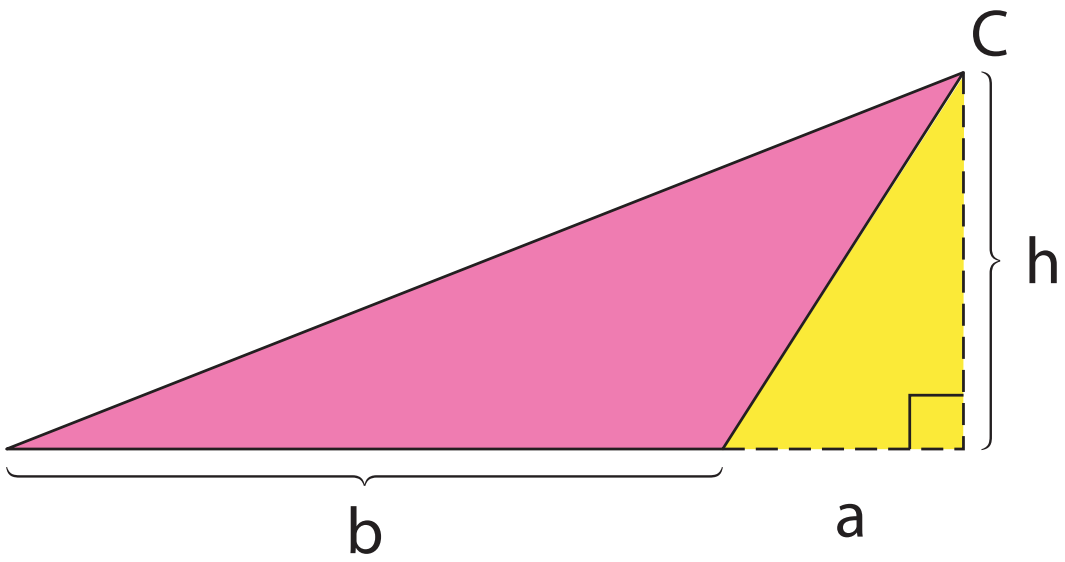
The area formula for triangles

Is the area formula still true for *this* base b and height h?

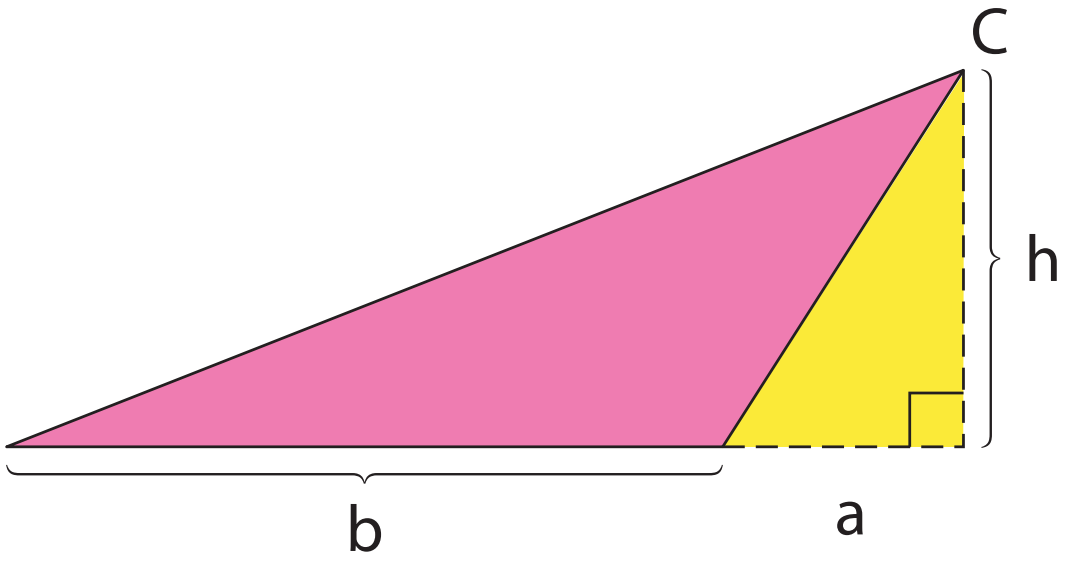


The area formula for triangles



We can view the triangle as a “difference” of right triangles.

The area formula for triangles



$$\begin{aligned}
\text{Area} &= \frac{1}{2} \cdot (b + a) \cdot h - \frac{1}{2} \cdot a \cdot h \\
&= \frac{1}{2} \cdot b \cdot h + \frac{1}{2} \cdot a \cdot h - \frac{1}{2} \cdot a \cdot h \\
&= \frac{1}{2} \cdot b \cdot h
\end{aligned}$$

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Compare this reasoning with the previous reasoning

Problem: What is 45% of 120?

Student solution: Half of 120 is 60. Ten percent of 120 is 12, so 5% of 120 is half of that ten percent, which is 6. So the answer is 60 minus 6, which is 54.

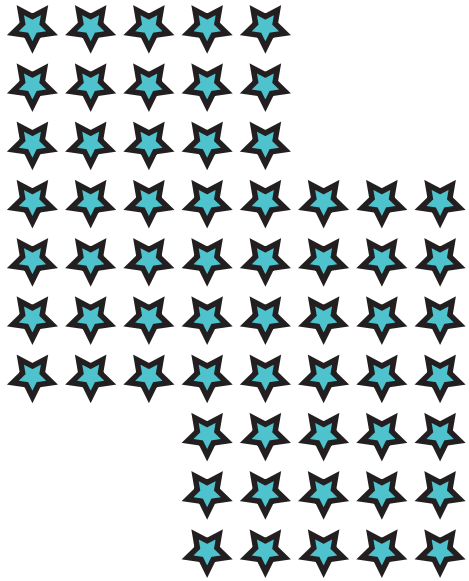
Write and interpret numerical expressions

Your expression:

_____

Explain this expression:

$10 \cdot 8 - 2 \cdot 3^2$



Explain this expression:

$2 \cdot (7 \cdot 5) - 4 \cdot 2$

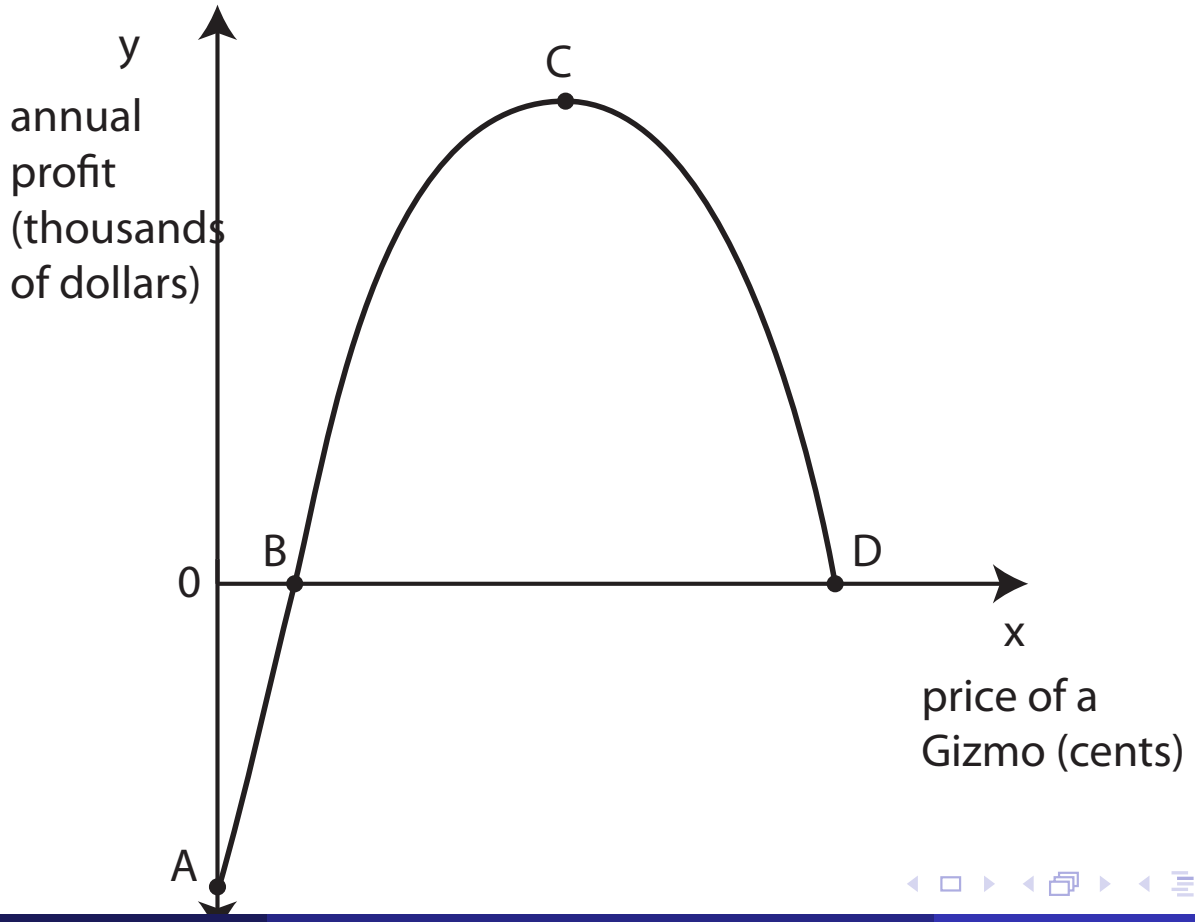
Seeing Structure in Expressions

Graph of the Gizmo Company's profit function

$y = 2(x - 10)(80 - x)$

$y = 2450 - 2(x - 45)^2$

$y = 2x^2 + 180x - 1600$



The Mathematics Teaching Community

Online at:

https
mathematicsteachingcommunity.math.uga.edu

or go to
www.math.uga.edu
find Sybilla Beckmann's webpage, and find the link there.

Thank you!
Questions? Comments?

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