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Improving mathematics instruction through lesson study: a theoretical model and North American case

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Abstract This article presents a theoretical model of lesson study, an approach to instructional improvement that originated in Japan. The theoretical model includes four lesson study features (investigation, planning, research lesson, and reflection) and three pathways through which lesson study improves instruction: changes in teachers' knowledge and beliefs; changes in professional community; and changes in teaching–learning resources. The model thus suggests that development of teachers' knowledge and professional community (not just improved lesson plans) are instructional improvement mechanisms within lesson study. The theoretical model is used to examine the "auditable trail" of data from a North American lesson study case, yielding evidence that the lesson study work affected each of the three pathways. We argue that the case provides an "existence proof" of the potential effectiveness of lesson study outside Japan. Limitations of the case are discussed, including (1) the nature of data available from the "auditable trail" and (2) generalizability to other lesson study efforts.

Keywords Lesson study · Professional learning · Professional development · Teacher change · Mathematics content knowledge · Pedagogical content knowledge · Teacher community

Introduction

Lesson study, the primary form of professional development in Japanese elementary schools, has spread rapidly in North America since 1999 (Lesson Study Research Group 2004). However, efforts to build a theoretical model of lesson study or to document the features and impact of lesson study have been modest to date (Lewis et al. 2006; Wang-Iverson and Yoshida 2005). Basic questions still need to be addressed, including: what are

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the features of lesson study; what are the mechanisms by which lesson study is posited to improve instruction; and what is the evidence that lesson study can be used effectively outside of Japan. Drawing on lesson study research (including Japanese sources) and on the broader research base on mathematics teachers' professional learning, we develop a theoretical model of lesson study and use it to examine the features and impact of a North American lesson study case.

The theoretical model of lesson study proposed in Fig. 1 draws on Japanese sources to describe lesson study features, shown in the left column (Hashimoto et al. 2003; Mukouyama 1999; Orihara 1997; Tsubota 2004). Lesson study is a system of collaborative learning from live instruction that uses the four features in the left column of Fig. 1—investigation, planning, research lesson, and reflection—to create changes in teachers' knowledge and beliefs, professional community, and teaching–learning resources, as shown in the center of Fig. 1 (see also Fernandez and Yoshida 2004).

A broader body of research on teachers' professional learning also informs Fig. 1. In particular, two major theoretical traditions inform the connection between lesson study features and intervening changes (Bowers et al. 1999). *Cognitive theories of teacher learning* conceive learning as changes in an individual's mental schemata, often in response to opportunities to *make one's own ideas visible* (for example, mathematical or pedagogical ideas) and to experience conflict between one's own ideas and ideas from colleagues, research, students, the curriculum, or other sources (e.g., Doerr and English 2006; Hashweh 2003; Lesh et al. 2000; Linn et al. 2004; Remillard and Bryans 2004). For example, teachers' examination of student work or teachers' joint examination of practice may surface ideas about mathematics and its teaching and learning that teachers use as a vantage point to examine and revise their own thinking (e.g., Chazan et al. 1998; Jacobs et al. 2007; Sherin 2002; Warfield et al. 2005).

Situated learning theories provide a second major theoretical underpinning for the intervening changes proposed in Fig. 1. *Situated learning theories* conceive learning as participation in a community that uses particular cultural "tools"—broadly defined to include norms, language, customary activities, external representations, and so forth. Participation in a community shapes the identity of members, and their future actions and commitments, and members transfer forms of participation (not simply skills) to new settings (Lave and Wenger 1991). As Lave (1991) writes, "Developing an identity as a member of a community and becoming knowledgeably skillful are part of the same process (p. 63)." Studies indicate that shared tools (such as curriculum and assessment) and individual and community characteristics (such as beliefs about students and structures to support collaborative work) mediate teacher learning (Borko et al. 2000; Collopy 2003; Lloyd 2002; Smith 2000; Spillane 2000). Research and theory on planned innovation further suggest that particular community norms support improvement, including: teachers' "ownership" of change efforts, their sense of responsibility to colleagues and students (as opposed to surface compliance with external mandates), and shared and negotiated practice (Ball 1996; Fullan 2001; Little 2003; McLaughlin and Talbert 2001).

In summary, the theoretical model posits that lesson study makes various types of knowledge more visible, such as colleagues' ideas about pedagogy and students' mathematical thinking, thereby enabling teachers to encounter new or different ideas, and to refine their knowledge, as cognitive theories propose. Second, the model posits that lesson study enables teachers to strengthen professional community, and to build the norms and tools needed for instructional improvement, as situated theories of learning propose. These might include norms of inquiry and accountability and shared language and frameworks for analysis of practice. Finally, the model proposes that improvement of teaching and

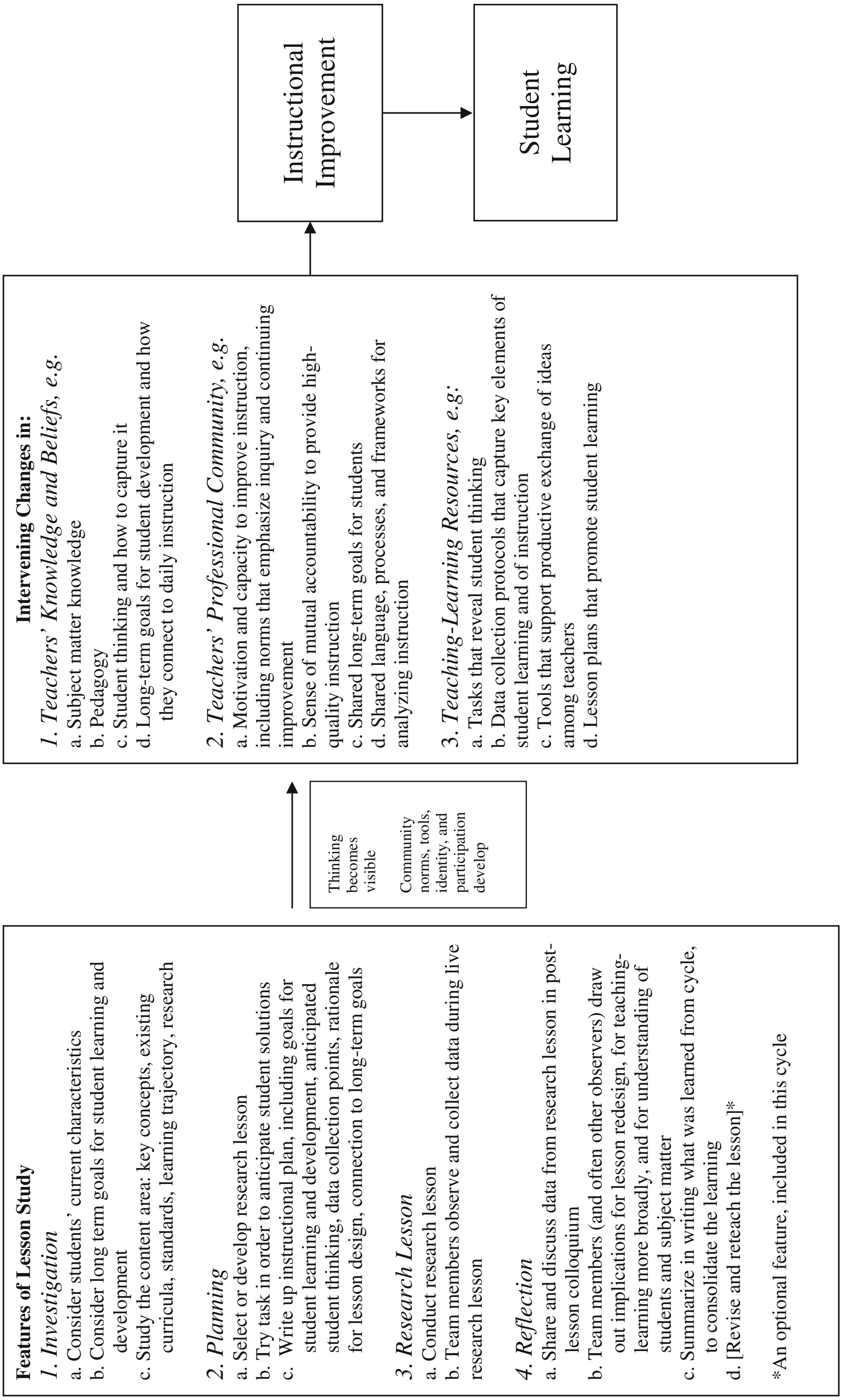


Fig. 1 How lesson study produces instructional improvement

learning resources (such as lesson plans and tasks) is a third pathway by which lesson study contributes to instructional improvement.

Because lesson study is a locally designed process, there is some variation in feature emphasis within the general theoretical model proposed in Fig. 1. For example, some lesson study groups may invest considerable time in investigation of teaching materials, called *kyouzai kenkyuu* in Japanese (Shimizu 1999; Takahashi et al. 2005), studying a range of teaching approaches and research resources, whereas other groups may simply use their existing textbook lesson to begin lesson study and instead invest more time in the latter part of the cycle, in which they analyze student responses and draw out the implications for lesson re-design. Whatever the local variation, the theoretical model in Fig. 1 suggests that lesson study groups use the features of lesson study shown in the left column to create the three broad types of intervening change shown in the center. We use the theoretical model to examine a North American lesson study case, investigating whether and how lesson study impacts teachers' knowledge and beliefs, professional community and teaching–learning resources.

Method

Description of the lesson study context

The lesson study cycle we describe occurred in August, 2002, as part of a 2-week summer workshop planned and led by mathematics teacher–coaches in a US school district. This cycle represents one small window on continuing lesson study work initiated and led by teachers and coaches in this district from 2000 through the present. This US K-8 public school district serves just over 10,000 students, of whom 29% participate in the free or reduced price lunch program. Race–ethnicity data indicate that 42% of students in the district are White, 28% Hispanic, 19% Asian, 6% Filipino and Pacific Islander, and 4% African American.

Focal group

Teachers in one lesson study group (six teachers from five different schools) were videotaped as they planned, taught, observed, revised, and re-taught a research lesson. Members of this group taught their research lesson twice during the summer workshop, and on each occasion one member taught the lesson and the remaining group members collected data later used to analyze and revise the lesson.

The group included four elementary teachers, one middle-school teacher, and a half-time elementary teacher/half-time mathematics coach ("Teacher 5") who co-founded the lesson study effort and is the third author of this article. Teacher 5 participated in a foundation-sponsored mathematics coaching program that had provided ~15–20 days per year of professional learning opportunities over the preceding 3 years. The foundation-sponsored meetings typically included opportunities to solve and discuss mathematics problems with fellow educators under the guidance of mathematically knowledgeable leaders.

Data analysis

Data for this case include: videotapes of all group meetings and both research lessons; written lesson plans; student work; researchers' fieldnotes; and records of follow-up conversations (by telephone or e-mail) with group members.

The first author reviewed all case materials and selected materials potentially relevant to the intervening changes in the theoretical model, compiling a series of video clips totaling about 3 h in length (from ~20 h of raw video) and an initial case draft, both of which were reviewed by the second and third authors who were asked to identify from the case materials both confirming and disconfirming evidence related to the intervening changes in Fig. 1.[1] The data presented below were nominated by all three authors as evidence of the intervening changes, and no contradictory data were nominated with respect to these changes. In order to further check on whether the initial reduction of video data might have missed data confirming or disconfirming the intervening changes, the written case and a video version of it were reviewed by the participants in the lesson study group. Their comments were minor and were incorporated into this case.

While much learning occurs invisibly, this report focuses on learning that left some visible trace—for example, through changes to a lesson or changes in verbalized understanding. The choice to focus only on this "auditable trail" (Lesh 2002, p. 27), rather than to include additional researcher-initiated assessments, was based on three considerations. First, use of the auditable trail required minimal intrusion by researchers into the teacher-designed and teacher-led lesson study cycle, and therefore minimal risk of reshaping the process or unwittingly contributing to teachers' learning through extra assessments. The primary intrusions into the lesson study cycle were the video camera and the presence of one observing researcher. Second, compared to the measures designed primarily for research purposes, the naturally occurring artifacts of lesson study, such as the publicly available video that corresponds with the transcripts provided below (Mills College Lesson Study Group 2005), are useful to other sites as "artifacts…that leverage learning" (Bannan-Ritland 2003, p. 24) about lesson study. Finally, standardized research tools (e.g., assessments of teachers' knowledge) are most useful when an intervention is relatively well specified and changes are expected to be uniform across teachers. Videotaping, fieldnotes, and artifact collection are more useful for an approach like lesson study that is not yet well understood and where individual teachers are expected to differ in what is learned (Zawojewski et al. 2008).

Lesson study group activities and rationale

The workshop was designed by the district mathematics teacher–coaches, including the third author of this article. Table 1 provides a detailed sequence of the activities of the workshop, the time devoted to each, and the posited link between each activity and the intervening changes shown in Fig. 1. As Table 1 shows, during the investigation phase of lesson study, teachers studied state mathematics standards and solved and discussed mathematics problems. During subsequent phases, they chose and slightly modified a research lesson, observed it and collected data as it was taught to students, and then discussed, revised, and re-taught it to another class. Hence, the final three features of the lesson study cycle—planning, research lesson, and reflection—were repeated. Reteaching is a common lesson study feature in the US that is an occasional feature of Japanese lesson study (Yoshida 2002). The two research lessons took place in two comparable grade 4 classrooms within a single elementary school, with workshop participants acting as "guest teachers" in the school.

[1] The first author was present during week-2 of the workshop as an observer of the lesson study group; the second author observed both weeks, but focused on a different lesson study group; the third author was a group participant and contributed to workshop design.

Table 1 Workshop activities linked to each lesson study feature and to theoretical model

Lesson study feature	Link to theoretical model
1. Investigation	
Solve algebra problems, share and discuss solutions, consider student thinking at different ages (*Days* 1–4; *Total* 7 h)	Build knowledge of mathematics, student thinking, colleagues' thinking
Study standards, curriculum, published lessons related to elementary foundations of algebra (*Days* 1–5; *Total* 4 h)	Connect daily instruction to long-term goals; foster knowledge of mathematics and pedagogy through analysis and comparison of existing lessons. Starting with existing lessons builds on existing resources
2. Planning	
Develop group norms, rotating roles, choose norm to monitor (*Day* 1; 1 h)	Norms and self-monitoring will build capacity for collaborative work. Role rotation distributes leadership, builds capacity
Select research lesson, do task and share solutions, anticipate student thinking, write instructional plan using template (goals, data collection points, connection to prior and subsequent learning, etc.) (*Days* 1–5, 6 h)	Choosing lesson builds teachers' buy-in. Doing task builds knowledge of mathematics and student thinking. Writing shared plan makes visible teachers' thinking about mathematics teaching–learning, surfaces differences. Instructional plan template focuses attention on typically neglected elements, e.g., long-term goals, student thinking
3. Research lesson	
One team member teaches research lesson planned by the group; others collect agreed-upon data (*Day* 6: 1 h, *Research Lesson* 1)	Observation of students is a key skill/habit for building knowledge. Observation may also build motivation to improve one's own teaching. A shared lesson provides a common referent, so teachers know what others mean by terms like "problem-solving"
4. Reflection	
Post-lesson discussion following protocol: (1) instructor's comments; (2) teammates' presentation of lesson data; (3) team panel discussion of agreed-upon questions; (4) audience comments; (5) invited commentator (*Day* 6: 1 h)	Protocol focuses discussion on student learning, thereby building useful, safe exchange of ideas. Involvement of outsiders increases team's access to content expertise
Cycle two: planning	
Revise instructional plan (*Day* 6,7: 4 h *total*)	Revision of lesson based on data can strengthen (1) habit of observing students; (2) attention to connections between lesson design and student thinking-behavior; (3) inquiry stance toward practice; (4) norm of instruction as something to be improved
Cycle two: research lesson	
Different team member teaches Research Lesson 2; others collect agreed-upon data (*Day* 8: 1 h)	Same as Research Lesson above, plus: Distributing teaching supports shared ownership, builds understanding of different roles
Cycle two: reflection	
Post-lesson discussion 2 (*Day* 8: 1 h)	Same as Post-Lesson Discussion 1, plus study of revised lesson builds sense of efficacy.
Final reflection to document learning from lesson study cycle (*Day* 8, 9: 2 h *total*)	Designated time and questions help refine and consolidate teachers' learning, build knowledge of colleagues' learning

Table 1 continued

Lesson study feature	Link to theoretical model
Observe research lessons planned by another lesson study group (*Days* 7, 9: 4 h *total*) and by Japanese teachers (*Days* 6, 10: 4 h *total*), participate in post-lesson discussions	Observation of other groups can suggest improvements to lesson study, catalyze reflection on own group's work. Experience with various roles (e.g., team member, audience member) increases understanding of desirable behavior within each role. Research lessons taught by accomplished peers build norm of career-long reflection even by "expert" teachers. Teachers from a different culture may provoke reflection on cultural assumptions about teaching–learning

Participation in the workshop was voluntary, and teachers received a modest honorarium. Since teachers gathered from different school sites in the district, the model is similar to the *district-based* lesson study that occurs in Japan (Murata and Takahashi 2002). Certain features of *school-based* lesson study (such as developing long-term goals for these students based on their current characteristics) are thus not prominent.

How the lesson study model unfolded

After studying mathematics standards and discussing how elementary mathematics can build understanding of algebra, the teachers chose to focus their work on the goal of helping students recognize and mathematically express patterns. For their research lesson, they chose a lesson from *Navigating Through Algebra in Grades* 3–5 (Cuevas and Yeatts 2001). The lesson task (including the problem context of seats and tables added by teachers) is summarized in Fig. 2. Figure 3 provides the student worksheet (which teachers took from the textbook, slightly modifying the heading titles). Team members solved and discussed the task themselves (using plastic triangles) in order to anticipate student solutions. As they did so, several difficulties in their own understanding of the mathematics surfaced as discussed below.

On workshop Day 6, Teacher 1 taught the research lesson to a fourth grade class while the remaining team members observed and collected data (mainly detailed narrative notes that "shadowed" selected students for the entire class period). In addition to the group members, seven teachers from another lesson study group, two visiting Japanese educators, and a researcher (first author) observed the lesson. Students were told about a "long, skinny classroom," with "triangle tables" arranged in a long row and were posed the following question: "Can patterns help us find an easy way to answer the question: How many seats fit around a row of triangle tables?" When asked to complete the chart shown in Fig. 3 (and a second, similar chart, where they could enter numbers of their own choosing), all 22 students completed the worksheets correctly.[2] However, when asked at the end of the lesson to explain the pattern in words or represent it as an equation, few students could explain the connection between the plus-two pattern and the problem, as the following excerpt from class-wide discussion reveals:

> Teacher 1: [asking the class to guide her in writing the equation on the board] The number of tables plus two equals what? [Teacher 1 pauses, sees only a few hands

[2] One of the 22 worksheets had a mistake in one row.

We have a long skinny room and triangle tables that we need to arrange in a row with their edges touching, as shown. Each side can hold one "seat," shown with a circle. Can patterns help us find an easy way to answer the question: How many seats fit around a row of triangle tables?

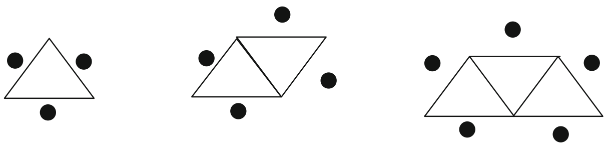
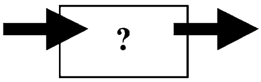


Fig. 2 Lesson problem and approximated board illustration

Triangle Rule Machine **Name** ________________

INPUT
Number of Δ tables ______

Rule



OUTPUT
Seats __________

INPUT Number of Triangle Tables	**OUTPUT Number of Seats**
1	3
2	4
3	
4	
5	
6	

What patterns do you see?____

Fig. 3 First worksheet, lesson 1

raised.] The number of tables plus two got us what? [Teacher 1 pauses again, still sees relatively few hands raised.] What are we trying to figure out? [Pause] What are we trying to figure out here? [Pause] We already know the number of tables. Jesse tells us we have to add two. What's that going to get us?… I'm not seeing many hands up… The number of tables plus two equals what?… [After a pause, Teacher 1 calls on a student with a raised hand.]
Student: Six.
Teacher 1: Now, where did we get six from? We don't know how many we started with.

Shortly after the lesson, a post-lesson discussion was held, attended by the lesson study team and the additional observers of the research lesson. The post-lesson discussion followed a structured agenda that included presentation of data collected by team members during the lesson and discussion of three questions formulated by the team, which focused on what students learned and what lesson elements supported learning or provided barriers. Team members pointed out that, despite uniformly correct worksheets, few students could describe the pattern in words (see transcript segment above) or in an equation. They also pointed out that the plus-one vertical pattern in the worksheet enabled students to fill it out simply by rote numbering down the column. Two ideas that emerged from the post-lesson discussion were that "Our worksheet set it up for them, spoon fed them" (Teacher 1) and that students might learn more *without* the worksheet, through "the messy business of organizing data" (Teacher 5).

Over the next day and a half, the lesson study team redesigned the lesson. Although the team eventually decided to eliminate the worksheet, this decision initially fell outside the comfort zone of some group members, including Teacher 3, a young teacher who had already agreed to teach the revised research lesson and did not like the idea of "throwing random numbers up on the board and letting the kids try to figure out stuff." Eventually the team agreed on three changes to the lesson. They decided to eliminate the worksheet and instead give each student a narrow strip of paper with a vertical line down the center. On the left of each strip was a particular number of tables and on the right a blank space to fill in the number of seats. They decided that students at a table would *not* have sequential numbers and that, after calculating the number of seats for their particular problem, students would share data at their tables and write a group poster about the patterns they discovered. In addition, they decided that, early in the lesson, students would be invited to share on the board their strategies for counting the perimeter units.

Two days after the first teaching of the research lesson, the revised version of the lesson was taught by a different member of the lesson study team to a different class of 22 fourth graders at the same elementary school, and once again the remaining team members and invited observers collected data. During the lesson, students shared their counting methods at the blackboard, including: counting all around; counting one for each triangle base and one more for each end of the figure; and counting two for the end triangles and one for every other triangle. The posters from five of six groups correctly described the plus-two pattern relating seats and tables (e.g., "there are two more seats than tables"). Since some groups wrote individually and some collectively, the precise number of students whose thinking is represented by the writing is not known, but it was between 7 and 13 (depending on whether the group scribe is assumed to represent only one or all group members' understanding in the cases of collective writing), compared to two students in the first lesson.

Once again, a structured post-lesson discussion followed the research lesson, focused on using the observational data to answer questions formulated by the team, including: "How did the changes from the first lesson to the second lesson affect the student learning?" and "Were the students able to see the plus-two pattern?" Observers reported that, compared to the prior lesson, more students grasped the plus-two pattern and understood its meaning and connection to the problem's geometry. As the lesson instructor (Teacher 3) said in her opening remarks: "This lesson probably got much more to the core of what we were trying to do…to really get the students to understand the pattern, understand the rule, as opposed to being able to plug in a number and get an answer in which we've spoon-fed it to them with the chart…."

After the discussion of the second lesson, the team members met again to reflect on and write about the lesson study cycle. They summarized in writing the changes made to the lesson, described what they had learned from revising and re-teaching the lesson, and responded to structured prompts, such as "From teaching this lesson twice and revising it, we learned that students______."

Results

Table 2 summarizes the data found in the auditable trail. Our theoretical model (Fig. 1) posits that three types of intervening change are produced by lesson study: changes in teachers' knowledge and beliefs; changes in professional community; and changes in teaching–learning resources. The auditable trail provides evidence of six changes in teachers' knowledge and beliefs about mathematics and its teaching–learning, including changes in teachers' understanding of mathematics, of student thinking, and of ideas about what constitutes good instruction. The auditable trail provides evidence of five changes in teachers' professional community, including development of norms related to inquiry and improvement of instruction, development of a shared goal, and development and use of improved processes for collegial work. Finally, with respect to the third type of intervening change, in teaching–learning resources, there is evidence of improvement of the lesson plan. This section examines each of the changes, organizing them by the three categories of intervening change.

Intervening changes: Teachers' knowledge and beliefs

Changes 1–6: *Mathematical and Pedagogical Knowledge.* Table 2 provides evidence on six changes in teachers' knowledge. The first three are changes in teachers' mathematical knowledge. These include increased ability to distinguish the problem's mathematical patterns; to discern whether the mathematical pattern would change if triangles were arranged differently; and to see a geometric reason for the $n + 2$ pattern (i.e., that the triangles each contribute one base and the two ends of the figure contribute the "extra" two). The fourth change listed in Table 2 pertains to knowledge of student thinking. Teacher 3 (and subsequently other teachers) noticed that students' counting revealed their thinking about the mathematical pattern; this insight was used to redesign the lesson, by asking students to share counting methods in the second teaching. Changes 5 and 6 relate to pedagogy: teachers learned that correctly completed charts do not guarantee student understanding of the mathematical pattern, and that students may learn more from organizing data themselves than filling in a pre-made chart. The power of eliminating the chart, and having students organize the data, was repeatedly mentioned by various team members as an important learning from the cycle. For example, in the post-lesson discussion following Lesson 2, Teacher 6 noted that the redesigned task challenged students to think more and also enabled them to take a more active role as learners: "As opposed to our first lesson, it wasn't just taught…. This time the students were doing the learning…. To me, as a teacher going back and thinking about lessons and lesson plans, that idea can be used anywhere—to make sure that *students* are the learners in the classroom." At the team's final reflection meeting, as team members listed all the changes to the lesson and what they had learned from them, they again revisited this issue:

> Teacher 5: So we pushed the kids more to explain their thinking [in the second lesson]
> Teacher 1: I didn't do that in the first lesson? I can't remember.
> Teacher 4: You did.
> Teacher 5: You did, but what we were asking them *about* was less substantial than the second lesson. ….In the second lesson things weren't in a sequential order, so that was a harder thing for them to think about…. The thinking we were asking them

Table 2 Summary of evidence from case

Type of change	Evidence
Teachers' knowledge and beliefs	
1. Distinguish recursive and functional patterns in the mathematics problem	On Day 3, T1 confuses the two patterns; by Day 5 she states them clearly and spontaneously[a]
2. See that arrangement of triangles makes a difference in triangle-perimeter function	Initially teachers suggest it does not make a difference how the triangles are arranged (T2, 3, 4) or are wondering (T1, 5); after solving and discussing, all agree it makes a difference
3. See the geometric reason for the "plus-two" numerical pattern	During Colloquium 1, T5 reports that a student's counting led her to see a physical reason for the $n + 2$ pattern, calling it "a big aha for me"
	In end-of-cycle reflection, T1 recalls T5's insight and contrasts her initial reaction to her current understanding "I did not see that as an important thing because I personally did not see the pattern that the ends are the plus-two"
4. See that student counting methods can reveal student thinking	T3 initially questions why it would be useful for students to share their counting methods; she later states that counting methods can reveal why there is an $n + 2$ pattern, and she has students share counting methods in research lesson 2
5. See that correct worksheets may not indicate learning	Teachers originally planned worksheet as test of student understanding of the plus-two pattern; after the lesson, teachers noted that some students correctly completed the worksheet without understanding the plus-two pattern: "We don't really know what they know"(all)
6. See that students learn from organizing data	Teachers revise lesson to eliminate chart. Several discussions contrast the "spoonfeeding" in the first lesson with "students do the work" in the second lesson (all)
Teachers' professional community	
7. Strengthened commitment to instructional improvement and ownership of improvement work	Group spontaneously decides to do another cycle of lesson study (after summer) (all)
	T3 initially focuses on external accountability ("what we have to do;" Day 2) but later focuses on accountability to students and to team members (Day 7)
	All teachers continue to participate in lesson study 6 years later[b]
8. Strengthened emphasis on inquiry	Reports looking forward to "tweaking" lesson and "watching and seeing what they do" (T1)
	Reports enjoying discovery that triangle placement matters (T1)
	Reports change in attitude toward textbook: "So that's a really good lesson for us; just because it is in there [textbook]…is it really the most effective?" (T1)
9. Increased sense of responsibility to colleagues and students	T3 initially unwilling to eliminate worksheet (says that managing students' "random" ideas is beyond her comfort zone) but later says she will eliminate it if teammates can convince her this will be best for students

Table 2 continued

Type of change	Evidence
10. Shared long-term goal	T5 suggests "curiosity about mathematical patterns," as a long-term goal, evoking disagreement by a member who suggests that goals must be measurable. After discussion, team members agree to make curiosity a long-term goal of the instructional plan
11. Improved processes for collegial work	During daily monitoring of the norm "sticking to the process" team members express frustration. Group agrees to change rotating facilitator's role to mark decisions and make sure all team members agree; at a subsequent meeting, the new facilitator does this (all)
Teaching–learning resources	
12. Revised lesson plan revealed and promoted student thinking better than original plan	2 of 22 students in lesson 1 wrote about the plus-two pattern relating seats and tables; 7–13 of 22 students did so in lesson 2

[a] Evidence supporting this change can be found in (Lewis et al. 2006, p. 14)

[b] This excepts one teacher who has left teaching to raise children and one who is deceased

> to do [in the first lesson] was not so hard. So when you asked them to explain their thinking
> Teacher 6: (In sing-song voice) Plus two, plus two. (Everyone laughs.)
> Teacher 1: That's right.
> Teacher 6: It was neater…It was pretty neat and clean.
> Teacher 5: We're so good at that.
> Teacher 6: I always thought I loved it a lot until I watched the students' results and I thought ooooh (deliberately grimaces) you'd better get used to a different style. (Everyone laughs.)
> Teacher 5: So we did make them think more.
> Teacher 1: Absolutely.

Intervening changes: Teachers' professional community

The data in Table 2 also suggest that teachers strengthened elements of their professional community, including their ownership of the improvement effort, inquiry stance, sense of responsibility to colleagues and students, shared long-term goals, and tools for collegial work.

Changes 7–9: Strengthened norms of improvement, inquiry, and responsibility. Several pieces of data, summarized in Table 2, suggest that the lesson study work built norms of commitment to instructional improvement. The lesson study team, on their own initiative, decided to meet again in the Fall to further revise the lesson and to teach the newest version of the lesson as a research lesson, in order to provide an opportunity for colleagues new to lesson study to practice lesson study data collection live (instead of practicing using a video, as this team had). Team members had noticed during Research Lesson 2 some additional improvements they wanted to make (for example, having students within a group negotiate a common understanding before writing on the poster, so that students would have more opportunity to refine their thinking). Teacher 3 taught the further revised version to her own class in the Fall, with team members and attendees at a district lesson study workshop observing and collecting data.

Following the summer workshop, team members also voluntarily participated in lesson study groups at their schools, and several team members further tested the idea that the worksheet short-circuited students' thinking. For example, these teachers conducted experiments in which they gave half of their students a chart like that found in Lesson I (and the other half no chart) when solving function problems. Since all the teachers originally volunteered for the summer workshop, it is hard to say whether their continued participation in lesson study and in inquiry after the workshop indicates *increased* motivation to improve. However, it worth noting that all team members who still teach still participate voluntarily in lesson study 6 years later.

Transcript excerpts in Table 2 also suggest development of an inquiry stance on the part of the teachers, supported by lesson study features including observation of students, reflection, and lesson revision. For example, during the first post-lesson discussion, after talking about whether to eliminate the worksheet so students would not be cued by a vertical pattern, Teacher 1 commented with evident enjoyment: "I'm thinking how would the lesson have been changed if we'd started off with 10 and then 17 and just these random numbers. It is a kind of fun to think about all the different things you can tweak, and then watch, and see what they do. Gee, I guess that is what you call lesson study!" (Teacher 1 laughs, joined by other team members.) Teacher 1's recognition that the task presented in the research-based textbook was not optimally designed leads her to publicly question their reliance on the textbook: "Why did we do this [vertical] pattern? Because it is in there [the textbook]. So that's a really good lesson for us. Just because it is in there…is it really the most effective?" As she later reflects aloud on her changed understanding of the mathematics, Teacher 1 suggests that her ideas about what it means to be a good teacher are evolving: "So it just shows that in all this math, well, in everything we teach, we're only as effective as our own level of understanding. So we have to keep pushing ourselves to delve into…the why, the how come…"[3]

Another area of development in professional norms suggested by the transcripts is teachers' expressions of accountability to colleagues and students, rather than to external mandates. For example, in their early work together (Day 2), Teacher 3 frames the work as something imposed by the outside: "Is our goal to fulfill that standard that's been circled for us? Is that what we have to do?" However, as they work together, team members increasingly frame their work in terms of their responsibilities to one another and to the students. Teacher 3, who had just 2 years of teaching experience, initially expressed discomfort with the idea of getting rid of the worksheets, but she underlined her willingness to "be convinced" by teammates about what was best for students:

> Teacher 1: I really like the idea [of eliminating the worksheets]…but I really feel that Teacher 3 is teaching this lesson on Wednesday. We can have terrific ideas. But we've always said that this has to be doable for us here in the United States. And it is got to be doable by the teacher. We're throwing out ideas. But you (looking at Teacher 3) have to feel comfortable with the lesson. It is fair to give you permission to say "I don't feel comfortable with that."…
> Teacher 3: …I think just throwing random numbers up on the board and letting the kids try to figure out stuff, I am just like oh good gosh I don't want to do it….
> Teacher 5: …(relaying suggestion from Japanese educator) He said what about if you really just pick one thing that you wanted to change about the lesson….

[3] Full transcript evidence supporting this change can be found in (reference omitted, Appendix, p. 14, August 14).

Teacher 3: My question is this. I still want to go back to the point of: I want to do this the most effective way, to where we can get them to understand what the rule is of 'plus two.'

Teacher 5: What it means.

Teacher 3: Yes, what it means. I want to make sure that really focusing on all these other things [eliminating worksheet, providing non-sequential numbers] will make it happen. I'm not saying it won't. But I need to be convinced myself. I need some convincing that that's the most effective way to do this.

In these quotes, Teacher 3 is shifting from talking about professional learning as fulfilling someone else's requirements ("what we have to do") to designing a lesson with maximum benefit for students, even if this falls outside her comfort zone. Willingness to try teaching outside one's comfort zone may be supported by the structure of lesson study, in which team members share responsibility for the lesson, so that it is "our" lesson not "your" lesson.

Change 10: Shared long-term goal. As they wrote the goals for the instructional plan, team members considered the lesson's connection to their long-term goals for students.

Teacher 5: I have one more little thing to add to the goals.

Teacher 1: (Laughing, pretending to hit her over head) Dang!

Teacher 5: Be curious about discovering patterns and representing them as rules. So not that they would just do it one time but that they would want to do it again. I want them to be curious.

Teacher 3: How would you evaluate that?

Teacher 2. Yeah.

Teacher 5: We'll talk about that when we talk about evaluating the lesson.

Teacher 3: OK.

The teachers then all agreed to include the fifth goal (the long-term goal of curiosity about mathematical patterns) in their instructional plan. They did not dismiss the goal simply because it was not obvious how to evaluate it. Their lesson study materials emphasized the importance of long-term goals (as well as goals for the specific lesson), bumping up against a belief on the part of at least one teacher that any goals they developed must be immediately measurable.

Change 11: Improved processes for collegial work. Evidence from the group's norm-setting and norm-monitoring suggests that the teachers also improved their processes for working together. From the beginning, the team used the collaborative structures introduced at the workshop, including: selection of a rotating facilitator; dual recording on chart paper and computer (in order to facilitate both discussion and preservation of ideas); group norm-setting; and selection of one norm to monitor at each meeting and self-assess at the end of the meeting. On Day 2, group members assessed the norm they had chosen to monitor that day: "sticking to the process." Teacher 2 voiced frustration that many ideas were discussed without coming to clear decisions on them, and Teacher 3 suggested that the facilitator might need to take a stronger role. Two other teachers suggested that the facilitator needed to "make sure that everybody's…opinion is counted" and "be a little more aggressive…poll everybody…say what we are doing right now." The following day, when Teacher 5 (the teacher–coach) began to segue into a new topic of conversation, the new rotating facilitator (Teacher 1) stopped her and implemented the active facilitation agreed upon the prior day, polling all group members to ask "Is everybody on board with this?" The group thus negotiated and then applied a new shared process for effective group functioning.

Intervening changes: Teaching–learning resources

Change 12. Improved lesson task and plan. The revised lesson plan likely represents an advance over the original lesson plan, both in the sense that more students could articulate the plus-two pattern relating triangle tables to seats, and in the sense that the revised plan provided a more “thought-revealing task” that enabled fellow students and observing teachers to better see student thinking as students compared, discussed, and wrote about their patterns (Lesh et al. 2000). Research by Schliemann, Goodrow, and Lara-Roth (2001) provides independent confirmation of the team’s finding that elementary students tend to look at the columns of a function table in isolation from each other, and that task redesign to present number pairs in random order helps students “move from computations to generalizations about how two sets of values were interrelated” (p. 4). It is possible that other resources for teaching–learning were improved during this cycle, since teachers actively talked about the post-lesson discussion agenda (i.e., the usefulness of the various parts including the data reports, team-generated questions, and outside commentator), strategies for data collection in the classroom, and strategies for communicating what they learned to others (e.g., the usefulness of the final meeting in which they recorded their learning). However, multiple lesson study cycles would be needed to assess whether other teaching–learning resources, such as protocols for discussion and data collection, were improved.

Discussion

In summary, the data in Table 2 provide evidence of the three types of intervening changes posited in Fig. 1. With respect to teachers’ content knowledge, the case indicates changes in teachers’ understanding of the mathematics of this problem and of alternate problems that students might create. With respect to pedagogical knowledge, both the lesson redesign and the transcript evidence indicate that team members changed their ideas about what constituted “understanding” by students and about the role of data organization in helping students learn. Since the study focused only on learning that left an “auditable trail,” we do not have a full picture of what each team member learned, or failed to learn, during this lesson study cycle. For example, we do not know whether every member of the team saw the connection between the geometric and numerical “plus-two” patterns, although it is clear that some team members gained this knowledge. While repeated individual assessment could have provided a clearer picture of each team member’s learning, it might also have altered the teacher-led character of the work and have augmented teachers’ opportunities to learn the mathematics under consideration. Neither do we know whether the changes in mathematical understanding verbalized by the participants will endure or impact future instruction, although it seems that, at the very least, the lesson study work created a “knowledge integration environment” (Linn et al. 2004) in which teachers were actively building their understanding by gathering data, making connections among different data sources, surfacing competing ideas, and making sense of contradictions (such as the correct worksheets and difficulty verbalizing the pattern). Table 2 indicates that the lesson study cycle provided opportunities for ideas from the curriculum, colleagues, and students to become visible, perturbing teachers’ initial thinking about the mathematics and its teaching–learning, and leading to revisions.

Although the period of the case was brief, there is some suggestion that the professional community also evolved. For example, the group honed its processes for working together after monitoring group effectiveness. Group members voluntarily decided to further revise and re-teach the lesson after the end of the workshop, and they have continued to participate actively in lesson study during the subsequent 6 years, suggesting some validity to their self-reports of enjoying the inquiry and finding it useful.

With respect to the third type of change, in teaching–learning resources, student data suggest that the lesson plan did improve, since the number of students who correctly wrote about the plus-two pattern relating tables and seats increased across lessons. It is possible that something other than the lesson redesign caused the change in student responses across these two lessons, taught to two comparable fourth grade classes in the same school at the beginning of the school year. However, no competing explanation was offered by the 15 lesson observers.

Several limitations of the case data should be noted. The auditable trail provides only a limited window on the learning that occurs in lesson study. So, for example, while the transcript data indicate that Teachers 1, 3 and 5 learned about the physical reason for the plus-two pattern, the learning of the other teachers cannot be confirmed or disconfirmed from the verbal record.

A second limitation of the case is the focus on a single lesson study cycle and therefore on short-term changes such as refinement of the lesson plan and changes in teachers' knowledge related to the lesson task. In theory, the intervening changes in Fig. 1 should build gradually over multiple lesson study cycles, as teachers build the knowledge, beliefs, identity, skills, norms, and tools needed to work together more effectively. A long-term study of lesson study in this district provides some evidence that this is indeed the case (Lewis et al. 2006; Perry and Lewis 2008).

Third, we do not know whether the changes found in this cycle represent long-lasting changes in teachers' knowledge and habits of mind that can be transferred to other instructional situations. While the statements posited as evidence of enjoyment of inquiry, sense of ownership, and so forth have face validity, the predictive validity of the statements is unknown**.** Professional development is a black box that is just beginning to be unpacked (Borasi and Fonzi 2002; Garet et al. 2001; Heck et al. 2008; TE-MAT 2008) and our fledgling effort to conceptualize and measure the intervening changes in lesson study needs future work. One direction this work could take is to develop reliable, predictive measures of the posited intervening changes (e.g., Center for Research on Context 2002; Hill et al. 2008). A problem that will be encountered, however, is that lesson study is shaped by the participating teachers, so that researchers cannot necessarily specify in advance the appropriate outcomes to test. For example, the teachers in this case chose the lesson task and inquired into questions of particular interest to them—such as the impact of the worksheet on student learning, the mathematics of alternate arrangements of the triangles, and the public sharing of student counting methods to help other students gain access to the problem. Although we think there is persuasive evidence that teachers developed several kinds of knowledge during this lesson study cycle, it would have been difficult to specify this knowledge in advance. Other mathematics education researchers argue that the knowledge base and expertise of teachers can develop in ways that can be seen as continually better even without a specified endpoint in mind (Doerr and Lesh 2003; Lesh and Kelly 1997; Zawojewski et al. 2008). Although the wide latitude within lesson study for teachers to shape their own learning may be important to developing ownership of the improvement effort and an inquiry stance (Fullan 2001; Steinberg et al. 2004), it makes use of pre-specified measures difficult.

It is reasonable, also, to ask about the efficiency of lesson study, compared with other professional learning approaches. Teachers invested about 17 h in planning, teaching/

observing, revising, reteaching, and reflecting on the two research lessons. Table 2 suggests, however, that the revised lesson plan constitutes a relatively modest part of what is gained from lesson study. Most of the developments in Table 2 pertain to development of teachers' own mathematical knowledge, pedagogical knowledge, inquiry stance, and professional norms related to instructional improvement. Hence, it seems quite important *not* to regard the primary benefit of the 17-h investment as the improved lesson plan. Rather, the lesson planning might be regarded as a vehicle to develop teachers' knowledge, beliefs, and professional community. Viewed in that light, lesson study may turn out to be an efficient investment of resources, particularly if teachers voluntarily continue to participate for many years, as the members of this team have done. The only major expenditures associated with this lesson study work were modest stipends for the participating teachers and for the teacher–coaches who planned the workshop. The lesson study work leveraged the foundation-sponsored professional development that the mathematics teacher–coaches had received, by providing a forum for teacher–coaches and teachers to work together around instruction.

Another important question concerns the generalizability of the case. To what extent might we expect other lesson study groups to achieve similar kinds of learning? Three features may distinguish this case from other lesson study cases. First, the group included a teacher–coach who had participated in a foundation-sponsored mathematics coaching program for 3 years. In other respects, her background may be more typical, in that she had no college or graduate courses in mathematics. As the transcripts reveal, she played important roles in several elements of group learning: she repeatedly suggested that the group members take up the triangles and solve the problem themselves; she helped the group avoid premature, incorrect conclusions about whether the arrangement of triangles mattered; she noticed how students' counting reflected their thinking; and she suggested that the group add the goal that students should become curious about patterns. Lesson study groups may need someone sufficiently experienced in mathematics learning to ensure such opportunities arise and are used productively. In the case under study, the individual was a classroom teacher with extensive mathematics professional development experience, although not with university-based mathematics training.

A second distinguishing feature is this site's collaboration with Japanese mathematics educators. Both during the prior summer's workshop and the workshop under study, experienced Japanese teachers taught public research lessons and observed and commented on lessons taught by the local teachers (Mills College Lesson Study Group 2003b). At the request of the lesson study group, one Japanese elementary teacher joined them for about 1 h of the lesson revision process, and expressed support (though he did not originate) the idea of eliminating the worksheet. Hence, the group had "on-call" access to mathematics teaching expertise that may not be available in some lesson study settings. Third, the site has collaborated with US lesson study researchers, whose support has included yearly theory-of-action interviewing, documentation of local activities, advice about lesson study when requested, and joint planning of the prior year's summer workshop.

In other respects, these teachers may face circumstances similar to those in many other US school districts. Neither time nor funding is designated for lesson study by their district, so they have spent time securing a patchwork of funding for teacher stipends and substitute teachers. Their own knowledge of mathematics, including that of the teacher–coach, is still evolving, as the transcripts illustrate. The Japanese and US mathematics lesson study resources on which they drew are now widely available to other groups in the form of video and print materials (Lewis 2002; Mills College Lesson Study Group 2003a, b).

Conclusion

This article proposes a model of lesson study features and intervening changes. The model argues that lesson study improves instruction by developing teachers' knowledge (of content, pedagogy, and student thinking), by building teacher professional community, and by improving teaching materials. The model is used to examine data from a US lesson study case. The case data indicate that teachers used lesson study to build their knowledge of mathematics and its teaching, their capacity for joint work, and the quality of the teaching materials, thereby providing an "existence proof" for the effectiveness of lesson study outside Japan.

It has been noted that teaching is "a cultural activity" and that "Teaching can only change the way cultures change: gradually, steadily, over time as small changes are made…" (Hiebert and Stigler 2004, p. 13). This case explores how a small, willing group of teachers used lesson study to make some of those "small changes"—to a textbook lesson, to their own thinking about mathematics and its teaching, and to their collaborative practice. Whether or not the small changes seen in this case will add up to long-term change in teaching culture cannot be judged from a single cycle of lesson study. However, if Hiebert and Stigler are correct that teaching can only change gradually, through small changes, then it is essential for researchers both to scrutinize these small changes and to further build the theoretical model that tells us where to look for change.

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