

Lesson Plan for 3rd Grade Introduction to Area

For the lesson on December 5, 2012

At Cesar E. Chavez Multicultural Academic Center, Ms. Alcántara's Class

Instructor: Erendira Alcántara

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1. Title of the Lesson: Why a Standard Unit?

2. Brief description of the lesson

In this lesson, students will find the area of a shape using unspecified units. Through critical and engaging discussion, students will observe that they have arrived at different areas due to the relative size of the unspecified units and conclude that having standard units is necessary to properly communicate with others.

3. Goals of the Lesson

- Students will understand that area is the amount of space inside a shape.
- Students will understand that area is defined by the size of unit.
- Students will appreciate the importance of standard units when communicating the area of a space.

4. Relationship of the Unit to the Standards

2.G.1. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.



This unit



3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

(a.) A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.

(b.) A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

3.MD.6 Measure area by counting unit squares (square cm, square m, square in, square ft., and improvised units).

3.MD.7 Relate area to the operations of multiplication and addition.

(a.) Find the area of a rectangle with whole-number side lengths by tiling it.

5. Background and Rationale

According to the Common Core Standards, students must learn the concept of area in third grade. The standards outline the following key ideas for area:

- Recognize area as an attribute of plane figures and understand concepts of area measurement.



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- Measure area by counting unit squares in square centimeters, meters, inches, feet and improvised units.
- Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
- Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
- Tile to show a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$, and use area models to represent the distributive property in mathematical reasoning. Recognize that area is additive.
- Find areas of rectilinear figures by decomposing them into non-overlapping parts, applying this technique to solve real world problems.

We have observed that students have many misconceptions about area. Students often confuse area and perimeter. Since these concepts tend to be taught within the same unit, students often confuse one for the other. Another misconception is that students do not fully understand the concept of area as the space inside a shape. Students become overly dependent on the idea that area is simply the algorithm length multiplied by width ($l \times w$). In addition, students often do not understand that area is non-linear in that they will try to measure area of a shape with a ruler.

The focus of this area unit is to develop students' understanding that area is the space inside the shape. In this unit, students will directly and indirectly compare area. Students will use arbitrary units to quantify the area of shapes to determine which area is larger. Students will use the method of tiling to find the area of spaces and understand that when tiling, one uses the same unit, and the units do not overlap, leaving no gaps in between. Throughout the unit, students will continually express their ideas in a mathematical journal. The math journal will contain both ideas of student thinking and the summary points of each lesson. The use of the math journal will support students in communicating their thinking.

6. Research and *Kyozaikenkyu*

Last summer, our team had the privilege of observing a series of four lessons on the topic of area, taught by Bill Johnson as part of the Mills College "Teaching Through Problem Solving" Institute at Prieto Academy. During that week, we learned for the first time a common sequence for students' development of the area concept, which includes comparing the sizes of shapes using direct comparison, then indirect comparison, then arbitrary units, and finally standardized units. Before attending this Institute, our team had a general awareness of the importance of teaching measurement and area through arbitrary units, but we had never explicitly taught the rationale for why standardized units exist. Even more, it is probably safe to say that we taught arbitrary units and standard units interchangeably at times, rather than building a strong foundation in the particular area concept first and then introducing the standard units to be used. John A. Van de Walle, in *Elementary and Middle School Mathematics: Teaching Developmentally*, addresses this confusion:

Perhaps the biggest error in measurement instruction is the failure to recognize and separate two types of objectives: first, understanding the meaning and technique of measuring a particular attribute and, second, learning about the standard units commonly used to measure that attribute. These two objectives can be developed separately; when both objectives are attempted together, confusion is likely. (p. 394)

We also know that the concept of area is one of the four key areas for 3rd grade according to the Common Core State Standards. We found out that, starting in the early grades, the geometry and measurement domains of the CCSS focus on arbitrary units. An example of this, from 1st grade, is seeing how many triangles fit inside a larger shape. In 2nd grade, students are expected to split a rectangle into rows and columns of equal-sized squares and count the total number. Finally, in 3rd grade, students are expected to learn that squares with side lengths of 1 cm are called “one square unit”, and that the number of these units that fit inside the figure is equal to the area of that figure. This progression seems to fit with the recommendations of Van de Walle.

In some ways, the 3rd grade scope and sequence of Saxon Math dovetails nicely with what we have already learned from Van de Walle’s text as well as from the logic of the CCSS. Early lessons include tasks in which students have to decompose and recompose shapes, such as finding the number of equilateral triangles that will fit inside of a hexagon. This sets the groundwork for students to define area as the amount of space that fills a shape. It is reasonable that the students use triangles as arbitrary units, rather than being introduced too hastily to the idea of standard units. Meanwhile, as the area concept is revealed incrementally throughout the year, students are slowly introduced to the idea of counting the number of shaded spaces within a square grid. Students learn to count these shaded squares quite readily, but the task is decontextualized and any connection to the concept of area is not made until the end of the year.

Alongside our own curriculum, we also decided to take a closer look at Japanese curriculum for our *kyozaikenkyu*. In Tokyo Shoseki’s *Mathematics for Elementary School*, students are expected to compare different size rectangles through direct comparison, indirect comparison, and decomposing and recomposing the shapes. Finally, the “1 square centimeter” is introduced, and students can use this standard unit to determine the area of various figures, as well as make their own figures of a predetermined area. However, nowhere in the curriculum did we find a lesson objective explicitly devoted to having students identify why the existence of a standard unit might be necessary.

The sequence found in the Japanese curriculum was what we observed Bill Johnson teach at the Summer Institute and it is the inspiration for our unit sequence as well. During the lessons taught at the Institute, the “1 square centimeter” was introduced at the end of one of the lessons, and many participants thought this part may have been rushed or may have created confusion for the class. In addition, we discussed that kids might need more of a chance to understand a rationale for why standard units are necessary. We decided to add an additional lesson with the goal of helping students understand this concept. We came to an agreement that the introduction of the standard unit should be given its own lesson, and it should include a way of students concluding for themselves why the existence of a standard unit is necessary. This is the focus of our research lesson.

7. About the Unit and the Lesson

In this section, we will describe how the unit and research lesson are designed to help students meet Common Core State Standards 3.MD.5 and 3.MD.6 as well as the Standards for Mathematical Practice.

In Lesson 1, students begin to develop a foundation for the area concept. They are presented with two picnic blankets and are asked which is the larger blanket. Students discuss their ideas for how to prove which of the two is bigger. Eventually, students come to a consensus that members of the class can sit on

each blanket. The larger blanket will fit more students. The teacher summarizes the learning by defining area as covering the inside of a shape. This adheres to CCSS 3.MD.5 (“recognize area as an attribute of plane figures and understand concepts of area measurement”).

In Lesson 2, we present the students with a problem from Tokyo Shoseki’s *Mathematics for Elementary School*. In the problem, two students look at posters on a bulletin board and wonder which takes up more space. The posters are surrounded by an array of rectangular student drawings. Students were allowed to use any method they wanted to find which had the larger area, including cutting the posters and overlapping them, as well as cutting the photographs and using them as units to fill the posters. The teacher helps the students come to consensus that cutting the posters and overlapping them was not the most practical way of comparing their area. Instead, units of equal size can be used to quantify the area of these shapes. Thus, Lesson 2 is the first in a sequence of lessons that help students meet CCSS 3.MD.6 (“measure area by counting unit squares”).

Lesson 3 features the following problem in the Shoseki curriculum. Students are presented with a square and a rectangle. Both shapes have hash marks along their borders, which define where gridlines might be drawn in order to form square units. Students can use any method to compare the shapes. This lesson differs from Lesson 2 because students are challenged to visualize the square units that can fill the inside of the shape without any visual clues (such as the rectangular photographs in the previous lesson). It is also the first time that students are exposed to square units, and students begin to think about why it would be helpful that the units are squares.

In Lesson 4, we presented the students with a problem from the new version of *Mathematics for Elementary School*. Partners play multiple rounds of “rock-paper-scissors”. The winner of each round colors in a section on a game board. However, the sections are of different shapes and sizes. At the end of the game, the winner has the larger territory for his/her color. Then the students are presented with a game board with four different color territories and are asked to prove which player won. Even though each territory contains eight colored pieces, the areas differ because the sections themselves are different sizes. This lesson helps further students’ understanding that units must be of congruent in order to accurately define the area of a shape as well as compare the areas of different shapes.

Lesson 5 was adapted from a 3rd grade *Everyday Math* lesson. In our version, students are given three congruent rectangles and expected to find the area of each using different units: squares, rhombuses, and trapezoids. This provides helpful practice with tiling, which will be necessary for the research lesson. The teacher helps students reach a consensus that square units are ideal because, unlike the other polygons in this lesson, squares can fill the space of a figure precisely without overlapping or leaving gaps.

The goal of Lesson 6, our research lesson, is for students to understand the importance of standardized units (as described in “Flow of the Lesson”). It should be noted that the research lesson adheres to parts of CCSS 3.MD.6, in which specific standard units, such as square cm and square in, are mentioned. Also, we plan to add context to this lesson by later reading and discussing Rolf Myller’s *How Big is a Foot?* with our classes.

Our final lesson of the unit, Lesson 7, is also adapted from *Mathematics for Elementary School*. Students are given centimeter grid paper and asked to draw shapes that have a given area of 2 cm². From this lesson, students will discuss how shapes can be subdivided and decomposed, such that two triangular (or

rectangular) halves of a unit square still combine to equal one cm^2 , even though they do not take the form of a square.

In summation, our team would like to highlight a few of the Standards for Mathematical Practice that we feel are most relevant to this unit. We believe that these lessons challenge students to *construct viable arguments and critique the reasoning of others* (SMP3). For all lessons, students are given ample time to construct their own ideas and write them in their journals as well as talk to classmates about the reasonability of various solutions. They are then expected to reflect in their journals about their peers' ideas as well as sum up their own learning. We paid especially close attention to modeling these practices for our ELL students, as many of them find it more challenging to communicate their mathematical ideas.

We would also like to highlight that the lessons expect students to *attend to precision* (6). At the start of the unit, students describe the size of shapes in vague terms—such as describing how “tall” or “skinny” they are, as well as saying that one is “a little bit” bigger than the other. By the end of this sequence of lessons, students are not only expected to express the area of shapes in terms of a specific quantity, but they have had practice in how to find the areas of shapes by precisely tiling units. By Lesson 7, the students have learned that those units must be congruent, and that they must fill the entirety of a shape, without overlapping or leaving gaps of space.

8. Flow of the Unit

Lesson	Learning objective(s)	# of lesson periods
1	Area: Covering a shape	1
2	Direct/Indirect Comparison of Area	1
3	Arbitrary Units	1
4	Let's Compare Area of Different Shapes!	1
5	Which Unit Should We Use: Square, Rhombus, or Trapezoid?	1
6*	The Importance of Having a Standard Unit (research lesson)	1
7	Let's Draw Some Shapes Using cm^2 !	1

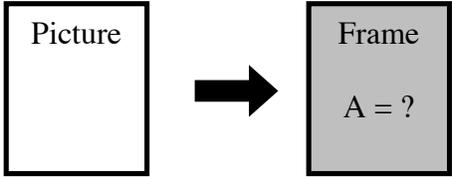
9. Flow of the Lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
1. Introduction Review student reflections from prior lesson. (5 minutes)		Can students recall what they learned the day before?
2. Posing the Task When you order pictures, you can order them in different sizes.	Show poster of identical picture in three different sizes. Show the picture frame.	

The holidays are coming and you bought a picture frame you want to give to your mom.

We need to order a picture that has the same area as this paper, so it will fit the frame.

Using your units, find the area needed to order the picture.



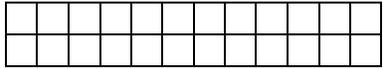
Demonstrate that the gray paper fits the frame.

Students paste scenario and problem statement into their journals.

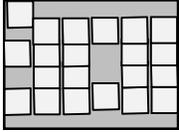
3. Anticipated Student Responses

- A) Different options of tiling
 - Will students tile in array form?
 - With 24 units squared, how could they know in what order the area is?
 - Why not 12" x 2", which is also 24 square inches.

E.g.1



E.g.2



- B) Why can't I use the one (the unit) I have?
(After all, they are all correct.)

- C) My unit's answer is correct because it is more efficient/uses less squares.

A) Students will be reminded that the units must cover the gray card in order to fit in the frame.

T: "Will the fit in the frame?"

- B) You can use that unit because it does cover the gray paper, but what if the photo technician's unit also covers the paper but is a different size than yours?
He would get a different number of square units.

- C) Remember, our goal is to focus on how to

- A) Are students tiling properly?
 - What method did they use?
 - Tiling properly?
 - Tiling entire area?
 - Tracing the unit to tile?
 - Is it in the form of a grid?

- B) Are students able to realize that the independent variable is the unit size?
 - "Which one do we use to communicate with the photo technician?"

- C) Are students aware that all the unit sizes could be

	<p>communicate our order to receive our photograph.</p>	<p>correct, but that they would have to communicate the size of the unit as well as the number of units in order to reach the correct size?</p>
<p>4. Comparing and Discussing</p> <p>1) Which group would like to share their solution first?</p> <p>2) Why did we get different areas?</p> <p>3) Since we know that the photo/gray paper and frame are the same, let's put them to the side. Let's look at the units. Can we all agree on one to make this easier on ourselves?</p> <p>4) When I want to order, how is the photo technician going to know which unit I am ordering in?</p>	<p>1a- Teacher will consciously select one group from each unit size to present to the class 1b- Students will bring up tiled gray paper and explain and label their answer (see Board Plan)</p> <p>2a- compare frame- no 2b- compare picture- no 2c- compare unit- yes!</p> <p>Students put everything away except for the units.</p> <p>Must communicate in same "language" using same unit-</p>	<p>Pinpoint student responses: 1a- Tiling properly? 1b- Grid 1c- Trace unit 1d- Tile unit</p> <p>Can students visualize multiple strategies to correctly find the area and hear other's explanations?</p> <p>Are students learning through discovery that, although all units can cover the same gray sheet, different sized units will yield a different answer?</p>
<p>5. Summing up</p> <p>Although all units can cover the gray paper, we must use the same one the photo technician uses to avoid misconceptions.</p> <p>A ruler is a tool that assures the unit is the same size.</p> <p>The number one is more/most efficient unit.</p> <p><Summary Points></p> <p>a. The area will change based on the unit size.</p> <p>b. You cannot predict the area of a figure just based on the square units. You also need the</p>	<p>Students are given an exit slip:</p> <p>Why do we need a standard unit?</p> <p>Where could standard units be useful in our lives?</p>	<p>Do students' answers consist of any of the following reasons why a standard unit is necessary?</p> <p>Possible Answers:</p> <ul style="list-style-type: none"> <input type="checkbox"/> Avoid misinterpretations <input type="checkbox"/> Avoid disagreements <input type="checkbox"/> Easy to work with <input type="checkbox"/> Simplify <input type="checkbox"/> Unify <input type="checkbox"/> Compare

size of the unit.

- c. We give a certain sized unit a name so we can identify it.

10. Evaluation

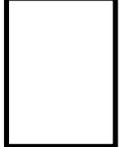
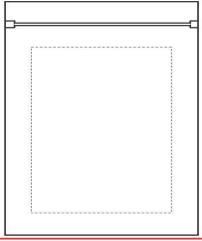
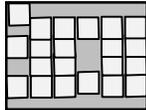
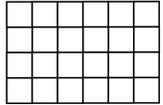
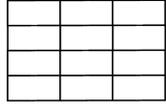
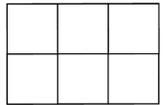
Name: _____ Date: _____

Write your ideas about these questions...

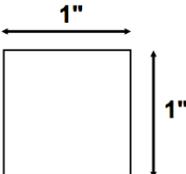
1) Why do we need standard units?

2) Where could standard units be useful in your life?

11. Board Plan

Review	Posing the Task	Anticipated Students Responses
<p>Students review reflections made in their math journals from the previous lesson.</p>	<p>1. Examples of Picture Sizes</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <i>Ex. 1</i> </div> <div style="text-align: center;">  <i>Ex. 2</i> </div> <div style="text-align: center;">  <i>Ex. 3</i> </div> </div> <p>2. Problem Statement</p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="border: 1px solid black; padding: 10px; text-align: center; width: 80px; height: 80px;">Picture</div> <div style="margin: 0 20px; font-size: 2em;">→</div> <div style="border: 1px solid black; padding: 10px; text-align: center; width: 80px; height: 80px; background-color: #cccccc;">Frame A = ?</div> </div> <p>3. Picture Frame With Unknown Area</p> <div style="text-align: center;">  </div> <p>4. Students Tile</p> <p><i>Units in bag</i></p>	<p><i>Examples:</i></p> <p>a.</p> <div style="text-align: center;">  <i>(Units not tiled properly.)</i> </div> <p>b.</p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="margin-right: 10px;">  </div> <div style="margin-right: 10px; font-size: 2em;">→</div> <div style="text-align: center;">  <i>4 units x 6 units</i> </div> </div> <p>c.</p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="margin-right: 10px;">  </div> <div style="margin-right: 10px; font-size: 2em;">→</div> <div style="text-align: center;">  <i>2 units x 3 units</i> </div> </div> <p>d.</p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="margin-right: 10px;">  </div> <div style="margin-right: 10px; font-size: 2em;">→</div> <div style="text-align: center;">  <i>4 rectangles x 3 rectangles</i> </div> </div>

Board Plan (continued)

Comparing & Discussing	Summary
<p>The teacher displays the different tiled frames on the board:</p> <p>Teacher: “Why did we get different areas?”</p> <p>Discussion Flow:</p> <ol style="list-style-type: none">1. Tiling Properly2. Compare area3. Compare units4. Which units should we use?5. Standardization of Units	<p><Realizations/Summary Points></p> <ul style="list-style-type: none">• The area will change based on the unit size.• You cannot predict the area of a figure just based on the square units. You also need the size of the unit.• We give a certain sized unit a name so we can identify it.• Introduction of standard square unit: 

12. Post-lesson Reflection