Public Research Lesson—English & Mathematics
Chicago Futabakai Japanese School
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1. Title of the lesson: Volume of a stair-step

2. Goals of the lesson: To reinforce student understanding of volume; to practice decomposing a shape into rectangular solids; to practice finding the volume of a rectangular solid; to practice communicating mathematical reasoning; to improve their English.

3. Relationship of the lesson to the Japanese Course of Study (from the c. 2004 translation of the 1989 Teaching Guide, 5th grade):

(7) The idea of volume and how to find it [B(2)]

(2) To enable children to understand the concept of volume and to measure volume in simple cases.

A. To understand the meaning of units of volume and of measurement
B. To know about the units (cubic centimeter (cm3) and cubic meter (m3)) to be used in measuring volume.
C. To know how to measure the volume of a cube and a rectangular prism.
D. To understand the meaning of capacity.

4. Discussion of the lesson

This is part English lesson, part mathematics. In the previous (math) class, students solved the problem shown in figure 1, in the Japanese textbook. The problem for today (figure 2) is taken from the English translation of 5A.

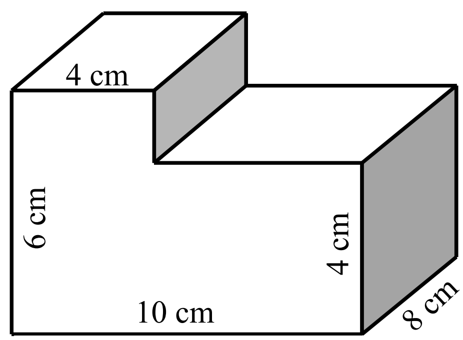


Figure 1: Previous problem

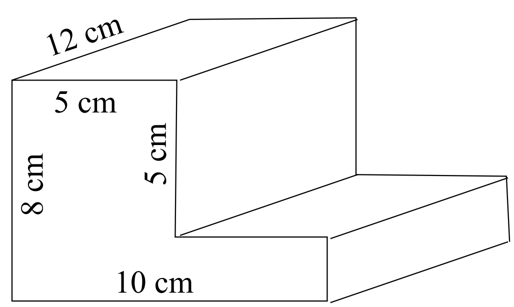


Figure 2: Today's problem

To facilitate communication and student thinking, we have made foam models of this shape, 2 copies of which will be provided to each group of students. Students will have the option of cutting the shape into component parts using dental floss.

Students solved the original problem by mentally cutting it into rectangular prisms in various ways, or by "filling in" the step to make a 6x10x8 box, then subtracting it. We expect those same methods to be used here. An additional method is possible for this shape: placing a second shape upside-down on top of it to form a box measuring 13x10x12. (See lesson plan, anticpated solution #5.)

We would like to open the possibility that some students might find the area of the hexagonal base (the front face in the figure 2) and multiply by the depth (12 cm). This powerful method generalizes to prisms with any shape base.

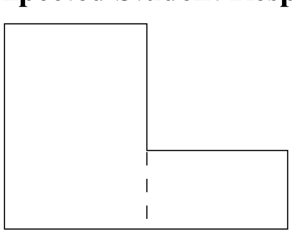
Students are more likely to think of this last method if they position the shape standing on a base and use cubes to build it. This goal led us to decide not to provide students with a picture of the shape, since the picture defines a particular orientation and may limit student thinking.

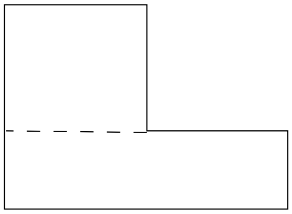
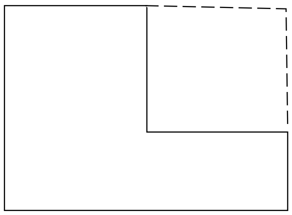
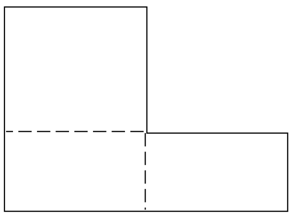
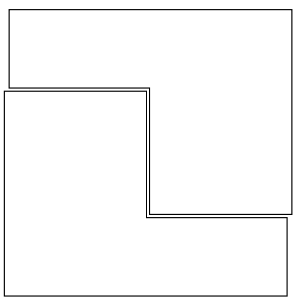
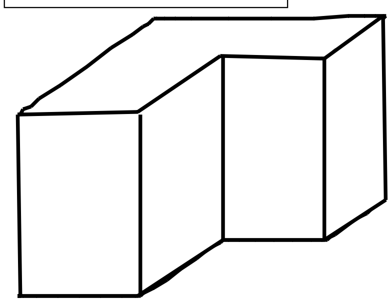
The problem then is how to convey measurement information, since that is normally done by the picture. We discussed whether to equip students with rulers, or to pre-mark measurements on the shape. We decided to provide a list of measurements and allow students to determine from the model where those measurements belong. The 1-cm cubes can be used to measure the figure if students are unsure.

In planning the discussion, since this is an English lesson, we will push all students to describe a part of their solution to the class. We will provide a vocabulary list to support them.

To encourage them to compare and contrast solutions, we will ask students to group similar solutions together.

Finally, to reinforce the concept of volume, we will ask students to consider why cutting a shape into parts is acceptable.

Steps, Learning Activities, Teacher's Questions, and Expected Student Responses	Teacher's Support	Points of Evaluation
Introduction (2 min.) Call me Mister Mac.	“Ohio gozimas!” Post a vocabulary list of English words: “volume,” “measurement,” “cube,” “box,” “shape,” “edge,” “face,” “width,” “length,” “height,” “add subtract multiply divide”	Is the vocabulary list helpful? Do students copy it?
Refer to prior work on volume, including the formula for volume of a box (refer to vocabulary list).		Do students recognize the formula?
Posing problem (5 min)		
“Before the holiday you found the volume of a shape like this. You found the volume in different ways. Today I would like to see what different ways you can use to find the volume of this shape.	Post: “Please think of different ways to find the volume of this shape.”	
“You probably need some measurements. I measured these when I was at home.”	Post the measurements: 12 cm, 10 cm, 8 cm, 5 cm, 3 cm.	
“I forgot to write down what parts of the shape had those measurements, but I think that’s OK:: I think you can tell where the measurements go. You can use these labels to record the measurements on the shape. So I am going to give each group two copies of the shape, a piece of paper with these measurements on it, and some labels. I'm also going to give you a bag of 1-cm cubes, in case you want to use them.”		
“You will also get a sheet of paper and a marker. Can you guess what for?” “If find more than one way, please use two sheets of paper.”		
“If you need something else, please ask.”		
“How long do you think you need to solve this problem?” ..... “Let's try 10 minutes.”		Does asking them this help motivate them to work efficiently?
Expected Student Responses (10 min.)		
 $5 \cdot 8 \cdot 12 = 480$ $5 \cdot 3 \cdot 12 = 180$ $180+480 = 660$ Ans: 660 cm^3	Provide dental floss if Ss want to cut their shape.	Are Ss clear about where the measurements go? Do Ss use the cubes to check?

Steps, Learning Activities, Teacher's Questions, and Expected Student Responses	Teacher's Support	Points of Evaluation
 $5 \cdot 5 \cdot 12 = 300$ $10 \cdot 3 \cdot 12 = 360$ $300+360 = 660$ Ans: 660 cm^3		Do students consider cutting the shape physically? Do the cubes help?
 $10 \cdot 8 \cdot 12 = 960$ $5 \cdot 5 \cdot 12 = 300$ $960-300 = 660$ Ans: 660 cm^3	Offer to provide a piece of foam for the “missing” piece.	
 $5 \cdot 5 \cdot 12 = 300$ $3 \cdot 5 \cdot 12 = 180$ $3 \cdot 5 \cdot 12 = 180$ $300+180+180=660$ Ans: 660 cm^3		
 $11 \cdot 10 \cdot 12 = 1320$ $1320 / 2 = 660$ Ans: 660 cm^3	This method is not likely. No need to push.	
 Area of base (55 cm^2) by a method analogous to 1-3, then: $55 \cdot 12 = 660$ Ans: 660 cm^3	We don't expect this method without some teacher intervention. See below.	
Students attempt to build the shape using the 1-cm cubes.	Provide hint card: “How could you build the shape with cubes so that each level (or layer) has the same number of cubes?”	Will Ss recognize that the shape can be seen as a stacking of congruent layers of cubes?
Ss might try multiplying three of the measurements, mis-applying the volume formula for a box, probably using the three longest measurements.	“How do you know which measurements to use?” Point to the concave part. “Does it matter how long these parts are?”	Do they learn from the other groups' presentations?

Steps, Learning Activities, Teacher's Questions, and Expected Student Responses	Teacher's Support	Points of Evaluation
	If no group attempts to build with cubes, suggest it (with hint card described above).	
Groups will be provided with a marker and large sheet of paper (11”x17”) and be asked to write their calculations.		
Discussion		
Each group posts all of its solutions on the board.		
Each group will present one of their methods. One student will be the “English helper”; the other students will be asked to describe part of their solution.		Do all students get a chance to practice English? Do groups make a point of presenting a method not used by previous groups? Do they note similarities to earlier solutions?
Ask them as class to organize their solutions—put similar solutions together.	“Did people have different ideas about how to organize the solutions?”	Can students identify similar solutions? Do they recognize that the two “cutting” solutions are similar?
Ask class to consider: Why is it OK to “cut” the figures into parts?		Can students articulate that volume remains constant even when the shape is cut?
End of lesson		

Post-lesson reflection:

(McDougal): My main concern with the lesson is we didn't pay enough attention to language issues—this was, after all, primarily supposed to be an English lesson using mathematics as the context. Students needed more support with vocabulary. One way to have handled this, which would have significantly lengthened the lesson, would have been to permit groups to identify the vocabulary they needed for their presentation, and make a class list of this vocabulary, in Japanese and English. This would also have been beyond what I could have done, but maybe some of the students would have been able to help.

Students did not use the cubes, although they seemed interested when I held them up as a tool.

No groups thought of the method of using (area of base) x (height). The hint card for this was inadequate.