8TH GRADE MATHEMATICS LESSON PLAN

April 26, 2007
Las Cruces, NM
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1. Title of the lesson: The Secret of The Crystal Ball
2. Goal of the lesson:
    1. To deepen students' understanding of the properties of the basic operations and place value by writing, interpreting, and using mathematical expressions through problem solving
    2. To help students become good problem solvers by
        i. encouraging them to use their prior knowledge to examine a problem situation in order to develop their ability to use logical reasoning to make conjectures, and
        ii. encouraging them to examine and justify the conjectures presented by their peers in order to find a solution to the problem.
    3. Provide opportunities for students to recognize the importance of working with their peers in order to deepen their understanding of mathematics
3. Instruction of the Lessons

In the Curriculum focal points for pre-kindergarten through grade 8 mathematics: a quest for coherence (National Council of Teachers of Mathematics Inc. Reston VA., 2006), one of the focal points in the middle school is to write, interpret, and use mathematical expressions and equations to solve problems. It is expected that students become able to:
    1) write mathematical expressions and equations that correspond to given situations,
    2) evaluate expressions, and
    3) use expressions and formulas to solve problems.

One of the challenges for the students is to write mathematical expressions that correspond to a given situation. Sometimes students may be reluctant to write mathematical expressions because they often try to find the answer by simply carrying out calculations and cannot see the merits of writing mathematical expressions. In order to overcome students' reluctance to write mathematical expressions, therefore, it is important that they learn how writing mathematical expressions can help them to solve problems.

When designing such problem-solving lesson, it is important to keep in mind that solving a problem is a process for providing an opportunity for students to appreciate that writing, interpreting, and using mathematical expressions. Therefore, the flow of the lesson should not solely focus on finding the correct answer, but also the process of solving the problem.

This lesson is designed for students' to understand how writing, interpreting, and using mathematical expressions help them analyze the problem situation and empower them to solve a problem.

The problem for this lesson to figure out the mechanism behind a trick named "Crystal Ball" from the website of a popular TV program, Ghost Whisperer (http://www.cbs.com/primetime/ghost_whisperer/crystal_ball.shtml). The website is based on a popular math trick and use Flash, multimedia authoring program for web applications, to make it interactive and engaging. The procedures that described on the website is

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Chose any two digit number, add together both digits and then subtract the total from your original number. When you have the final number look it up on the chart and find the relevant symbol. Concentrate on the symbol and when you have it clearly in your mind click on the Ghost Whisperer crystal ball and it will show you the symbol you are thinking of.....

In order to find out the trick, one of the approaches is to try several specific examples to find a pattern among the examples. Students typically use this inductive approach and find out that there might be mechanism behind the trick but it is difficult to figure out why the pattern exists. Another approach is to investigate the process of calculations described in the "Crystal Ball" instruction in order to find out what calculations are actually carried out to get the symbol that you need to imagine. This deductive approach demands that students write, interpret, and use mathematical expressions to investigate the trick, then find out why the crystal ball always gives you the same symbol no matter what two digit numbers are chosen. During this investigation, students will be using their previous learning of the properties of the basic operations, the notion of place value, and the use of symbols in mathematical expressions to see the generalized pattern.

    4)   Flow of the Lesson

Learning Activities, Teacher's Questions and Expected Students' Reactions	Teacher's Support	Points of Evaluation

1.   Introduction to the Problem
By experiencing the "Crystal Ball" on the internet, students will become familiar with the site.
   1. Chose any two digit number,
   2. Add together both digits,
   3. Subtract the total from your original number
   4. When you have the final number look it up on the chart and find the relevant symbol.
   5. Concentrate on the symbol and when you have it clearly in your mind
   6. Click on the crystal ball to see the symbol
	Ask a couple of volunteer students to try the website so that all the students understand the procedures described on the webpage. Help students to see the website always gives you the relevant symbol.	Do students understand the procedure?
		
		Do students see what is happening on the website?

2.   Posing the problem
By asking the following question, engage students to find the trick behind the "Crystal Ball" webpage.
With which opinion do you agree?
   a. It is just a coincident and there is nothing special in the "Crystal Ball" webpage.
   b. There might be a trick behind the "Crystal Ball".
   c. The "Crystal Ball" webpage actually reads your mind.
Let's find the trick behind the "Crystal Ball" webpage!
	Each student will be working with his/her partner to find a trick by using their prior knowledge.
	
	Provide students with worksheets to keep their work for the whole class discussion.
		Do students see there must be a trick behind the "Crystal Ball" webpage

3. Problem Solving
Working with a partner, students try to find the trick behind the "Crystal Ball" webpage.
Anticipated students' responses:
   a. Try a couple of specific examples to notice that the relevant symbol might be always the same but do not know why these symbols are the same.
   b. By examining several specific examples, he/she realizes that the final number will always be a multiple of nine, and the symbols on the chart that correspond to multiple of nine are all the same. However, he/she does not know why the final number will always be a multiple of nine.
   c. Write, interpret, and use mathematical expressions to investigate the trick
    · $a\,b$ as a chosen two digit number
    ·   The value of $a\,b$ is $10a+b$
    ·   Write a mathematical expression to express the procedure
     $(10a+b)-(a+b)$
     $=10a+b-a-b$
     $=10a-a+b-b$
     $=9a$
   Therefore the final number will always be a multiple of nine
	Encourage students to try at least a couple of specific examples.
	
	Help students understand that methods (a) and (b) may not be able to answer the question why all the final numbers give you the same symbol.
	
	Encourage students to investigate the process of calculations described in the instructions to "Crystal Ball" in order to find out what calculations are actually carried out to get the symbol that you need to imagine.
		Do students try at least a couple of specific examples to notice that the relevant symbol from your calculation might always be the same.

4. Discussing Students' Solutions (1) Ask students to explain their solutions to the other students in the class. (2) Facilitate students' discussion about their solutions, then lead them to understand that writing, interpreting, and using mathematical expressions helped them understand the trick behind the "Crystal Ball" webpage.	Write students' solutions and ideas on the blackboard in order to help students understand the discussion.	Can students explain their solutions to their peers? Can students examine and justify the solutions presented by their peers?
5. Summing up (1) Using the writing on the blackboard, review what students learned through the lesson. (2) Ask students to write a journal entry about what they learned through this lesson.	Encourage students to use the writing on the board as a reference when they write the journal entry.	

Reference

National Council of Teachers of Mathematics Inc. Reston VA. (2006). *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*.

GHOST WHISPERER
CRYSTAL BALL

Choose any two digit number, add together both digits and then subtract the total from your original number.*

When you have the final number look it up on the chart and find the relevant symbol. Concentrate on the symbol and when you have it clearly in your mind click on the **Ghost Whisperer** crystal ball and it will show you the symbol you are thinking of...

* For example if you chose 23: 2+3 = 5. 23 minus 5 will give you your answer.

99	79	59	39	19
98	78	58	38	18
97	77	57	37	17
96	76	56	36	16
95	75	55	35	15
94	74	54	34	14
93	73	53	33	13
92	72	52	32	12
91	71	51	31	11
90	70	50	30	10
89	69	49	29	9
88	68	48	28	8
87	67	47	27	7
86	66	46	26	6
85	65	45	25	5
84	64	44	24	4
83	63	43	23	3
82	62	42	22	2
81	61	41	21	1
80	60	40	20	0

http://www.cbs.com/primetime/ghost_whisperer/crystal_ball.shtml

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Board writing Plan for the "Crystal Ball"

The Crystal Ball
1. Chose any two digit number,
2. Add together both digits,
3. Subtract the total from your original number
4. When you have the final number look it up on the chart and find the relevant symbol
5. Concentrate on the symbol and when you have it clearly in your mind click on the crystal ball to see the symbol

What is happening on the website?
Use the worksheet to figure out

Case 1: 56
       5+6=11
       56-11=45   Symbol A
Case 2: 78
       7+8=15
       78-15=63   Symbol A

Students' approach A

Students' approach B

Students' approach C

The Crystal Ball always give you the same symbol no matter what two digit numbers are chosen because
1. the final number that the procedure give by the Crystal Ball always be a multiple of 9,
2. the symbols on the chart that correspond to multiple of 9 are all the same. (with two exceptions, 90 and 99)