9th Grade Algebra Lesson Plan

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1. Title of the lesson: Reciprocals of Quadratic Functions
2. Goal of the lesson:
   i. Students will be able to take all of their previous knowledge of relations, linear functions, quadratic functions and exponential functions to correctly graph an unfamiliar function - the reciprocal of a quadratic function.
      a. Given an equation, students will be able to identify the x-intercept(s) and y-intercept based on their knowledge that an x-intercept is when y = 0 and a y-intercept is when x = 0
      b. Given an equation, students will be able to identify the axis of symmetry based on their knowledge of the axis of symmetry in a quadratic.
      c. Given an equation or a graph, students will be able to identify what is happening as x approaches infinity and negative infinity. (The students have discussed end behavior when working with exponential growth and decay. However, our goal here is that students will be introduced to negligible numbers when x approaches infinity or negative infinity. For example, in $y = (x+3)^2$ the 3 is negligible when x approaches infinity.)

   ii. Given an equation, students will be able to identify discontinuity on the graph (specifically vertical asymptotes) based on the fact that it creates a number divided by zero which the students know is undefined.

   iii. To be able to make connections between the graph of $y = (x+3)^2$ and it's reciprocal, as shown in the table.

ANALYZE	$y = (x+3)^2$	$y = \frac{1}{(x+3)^2}$
axis of symmetry	x = -3	x = -3
behavior of graph near axis of symmetry	We see as x → -3, y → 0. Can look at x approaching -3 from above and below.	We see as x → -3, y → ∞. Can look at x approaching -3 from above and below.
end behavior	We see as x → ∞, y → ∞. Can look at x approaching -∞ too.	We see as x → ∞, y → 0. Can look at x approaching -∞ too.
horizontal asymptote	None	y = 0
y-intercept	(0, 9)	(0, 1/9)
x-intercept	x = -3	$0 \neq 1$ so there is no x-intercept
vertical asymptote	None	x = -3
graphs intersect	At x = -2 and x = -4 because that's where both functions have a y-value of 1. (These functions are reciprocals of one another and 1 and -1 are their own reciprocals.)	

3. About the unit

i. Name of the unit: Quadratic Functions (Extension relating the reciprocal functions)

Note: This lesson could also be used as an introduction to a unit on rational functions.

ii. Goals of the unit:

- Graphing quadratic functions using x-intercepts, y-intercepts, axis of symmetry, vertex and end behavior.
- Discussing the discriminant of the quadratic function and how it affects the graph.
- Determining a regression equation of data that can be modeled using quadratics, exponentials and linear functions.
- Graphing the reciprocal function of a quadratic using vertical and horizontal asymptotes as well as x-intercepts, y-intercepts, axis of symmetry, vertex and end behavior. To compare these graphs to the quadratic functions.

iii. Plan of the unit (Total 11 periods, including 2 review days, 1 quiz day and 1 test day)

1.1 Discriminant (1 period)
1.2 Graphing Quadratics (3 periods)
1.3 Regressions (2 periods)
1.4 Reciprocal Functions (1 period) – *this lesson*

4. Content connections to other units

Content goals of the lesson	**Prior work**
Create and explore the graph of a rational function.	Understanding of graphing linear, exponential and quadratic function as well as an understanding of x-intercepts, y-intercepts, end behavior and axis of symmetry.
Experience both vertical and horizontal asymptotes.	Understanding of end behavior and what happens as x approaches infinity with linear, exponential and quadratic function. Understanding that a number divided by zero is undefined.

5. Instruction of the Lessons

According to the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000), one of the major goals of number and operations in the 9-12 grade band is to develop a deeper understanding of very large and very small numbers and of various representations of them. Students need to recognize the difference between a negative exponent and a positive exponent in scientific notation and to understand that sometimes an ERROR can be a response for either an invalid operation or a number that overflows the capacity of the calculator.

Prior to this, the students have had the opportunity to discuss the end behavior and what happens when x becomes a very large positive number or a very large negative number, and will be able to connect that information to this graph. However, before this lesson, the students have not seen a graph that had discontinuity and will thus have to compare the values of y as x approaches the vertical asymptote in this function. The students will also be able to recall the relationships of reciprocal numbers and that 0 has an undefined reciprocal. The students will have to use approximations of numbers that can be calculated because they cannot calculate an expression containing infinity or a number that makes the denominator zero. The students will instead have to put in numbers approaching these numbers.

The *Principles and Standards* also says that 9-12 grade students should be able to understand patterns, relations and functions. Within this, students can understand and compare the properties of classes of functions, including exponential, polynomial and rational. The students have already worked with linear, exponential and quadratic functions and will use their knowledge to see how these relate to rational functions, specifically a reciprocal of a known quadratic. The students will be asked to generalize their findings and compare and contrast the two functions and their qualities.

> High school students' algebra experience should enable them to create and use tabular, symbolic, graphical, and verbal representations and to analyze and understand patterns, relations, and functions with more sophistication than in the middle grades. In helping high school students learn about the characteristics of particular classes of functions, teachers may find it helpful to compare and contrast situations that are modeled by functions from various classes.
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> National Council of Teachers of Mathematics, 2000

The students, while being asked to create a graph of an equation they have never seen before, must also use their problem solving skills. They need to identify the connections between the previous functions and build upon them to graph this function. As with many research lessons, at the end, the students will be asked to communicate their mathematical findings, not only within their collaborative group, but also to the remainder of the class.

Based on our previous knowledge of students and their understanding of graphing in general and specifically graphing rational functions, we found this lesson to be important. This lesson was initially planned as an introduction to rational functions for an Advanced Algebra curriculum - a lesson to view rational functions as familiar and pull away from the usual formulaic presentation. However, when creating the lesson, we realized that it also challenged a first year algebra student to synthesize all of the information they have learned throughout the year and use it with a new function. The new function being both familiar, as it is the reciprocal of a function they have previously graphed, and new in the fact that they have not had restricted domain before.

The function $y = \frac{1}{(x+3)^2}$ was chosen because the current unit is quadratics, so the reciprocal of a quadratic seemed appropriate. We chose to have the denominator be a factor that is squared because we want to introduce the concept of vertical asymptotes, but wanted to limit the graph to only having one vertical asymptote. We considered the functions $y = \frac{1}{x^2}$, $y = \frac{1}{\left(x \pm 1\right)^2}$, and $y = \frac{1}{\left(x \pm 2\right)^2}$, but not did think the vertical asymptotes were appropriate because the properties of 0, 1 and 2 have special characteristics that other numbers do not have. For example $2^2 = 2 \bullet 2 = 2 + 2$, where as 3, does not have this characteristic.

In order to facilitate the lesson and to motivate the students, the lesson will begin with a challenge set in an espionage environment. The warm-up is a review of graphing a quadratic function and identifying key characteristics. The second problem then introduces the concept of a reciprocal function. The students at Stevenson High School nearly always sit in groups of three to four students in their mathematics classroom. Much time has been devoted to teaching the students how to work effectively as a group. Throughout the year they have learned to communicate, collaborate, as well as practice coming to a consensus. The students will be asked to use these skills in this lesson by communicating within their group as well as to create a presentation to share with the rest of the class. Their groups have been assigned by the teacher and are changed every six weeks. At the time of this lesson the students will have been sitting with their current group members for the previous four weeks.

6. Flow of the Lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
1. Introduction to the Problem The ABI (The Algebra Bureau of Investigations) is considering hiring your team to create graphs. You have been recommended to the ABI because you have had so much experience graphing linear, exponential and quadratic functions. Before you will be fully accepted into the program you have been asked to graph and identify the key characteristics of $y = (x+3)^2$ as a test of your knowledge without the use of a calculator. Be sure you are as informative and as accurate as possible in displaying all you know about this function Complete your task to be hired and good luck!		
Anticipated Student Responses: 1. Students will correctly graph the function using a table of values with integer values of x. 2. Students will correctly graph the function $y = (x+3)^2$ using the x-intercept (-3, 0), the y-intercept (0, 9), the axis of symmetry x = -3, the vertex, also (-3, 0), the reflection of the y-intercept (-6, 9), the graph opens up, is normal width and end behavior as x approaches infinity or negative infinity, y approaches infinity based on their previous knowledge of quadratics. 3. Students will correctly graph some of the 8 characteristics in 1, but not all. a. Students will not list end behavior as a characteristic because they do not consider it necessary to graph b. Students will not list the reflection point as a characteristic because they do not consider it necessary to graph. 4. Students will incorrectly graph the reflection point at (-3, 9), (6, 9), or another incorrect point based on incorrect use of the axis of symmetry 5. Students will be confused because the vertex and the x-intercept are the same. 6. Students will incorrectly identify the vertex/x-intercept as (3, 0). 7. Students will incorrectly identify the y-intercept as (0, 6) because they double instead of square 3.	Give a large poster paper to each group and markers. Encourage groups to work independently of other groups and the teacher. Ask teams having difficulty what the important parts of the graph of a quadratic function are. After all groups finish working on the introductory problem, the teacher will ask for suggestions to put on a list of 'important parts of a graph'. The list will be on a large poster paper and will cover mostly the 6 characteristics listed in #2 on the left.	Do students understand the situation? Do students remember the important characteristics of the graph of a quadratic function? Are students able to identify the key characteristics of most graphs after graphing this quadratic function?

2. Problem Solving

Congratulations on your new position with the ABI. While we realize this may be unfamiliar to you, the ABI has requested that you graph the function $y = \frac{1}{(x+3)^2}$ using the knowledge you have from graphing all previous functions.
The ABI is already considering your team for promotion, and would like to see the quality of your work on graphing unfamiliar functions. However, it is most important that the graph be accurate, so if you do need help we have set up a source for you to get information from. You will meet this source (Code Name – Brown Eagle) at the secret rendezvous (Code Name – Escritorio de maestra) to ask any pertinent questions. Limit your visits to the rendezvous location to keep its location Top Secret.

Complete your graph and good luck!

Anticipated Student Responses:

1. Students will correctly graph the function using a table of values, being sure to look at the decimal values between -2 and -4.
2. Students will incompletely graph the function using a table of values only using integer values for x.
3. Students will correctly graph the function by finding each of the following
   a. End behavior by substituting in a large positive number and a large negative number for x.
   b. Axis of symmetry using what they know about quadratic functions
   c. y-intercept by substituting in x = 0.
   d. No x-intercepts
   e. Vertical asymptotes by finding that the function does not exist when x = -3 and does exist infinitely close to x = -3.
4. Students will incompletely graph the function by finding some, but not all of the 5 characteristics listed above.
5. Students incorrectly get confused with what happens as x = -3.
   a. Do not understand what happens here.
   b. Will think the x-intercept is (-3, 0)
   c. Will think the function does not exist from -2 to -4.

Encourage groups to work independently of other groups and the teacher.

If students need calculators, assign one student per group to be the calculator team member to encourage collaboration instead of separate working for each student.

Ask teams having difficulty to look at the list of important parts of the graph.

Advise students to use the tools they used in the warm-up and see the relationships between the first and second function.

After five minutes provide students with hint cards if they are still struggling.
- What can we use to graph an unknown function?
- What happens when x gets very large or small?
- What are the important points of most functions?
- What happens at x = -3?
- What happens between x = -2 and x=-4?

Do students understand the situation?

Do students remember the important characteristics of the graph of a function?

Are students able to work with non-integer values of x?

3. Extension Groups that correctly graph the function and are waiting for the rest of the class to finish will be given the following problem to help guide the classroom discussion in part 4. The ABI is thrilled with your diligence, and asks your team to assemble a report comparing and contrasting the two functions $y = (x+3)^2$ and $y = \frac{1}{(x+3)^2}$.	Encourage students to get up and look at the previous problems posted around the room.	
4. Discussing Students' Solutions a. Ask students to bring up their poster paper and explain their solutions to the other students in the class beginning with the group with the least accurate graph b. Have students who have the exact same characteristics of the graph to bring their poster paper up and place it next to the one already presented. c. Facilitate students' discussion and lead them to the relationships described in the table on page 1 by asking the students how this graph related to the graph of the quadratic. d. A table like the table on page 1 will be created to compare and contrast the characteristics of the two graphs.	Encourage questions for the group about their graph. Encourage students to say what qualities the presenting group's graph has that theirs also has – ask them to hold off on differences until they present.	Can students explain their solutions to their peers? Can students examine and justify the solutions presented by their peers?
5. Summary - Ask the students to "Stop and Jot" and speculate what they think the graph of $y = \frac{1}{(x-4)^2}$ would look like. If there is no time, ask them to journal about this for homework.	Encourage independent work without the aid of a calculator or group members. Encourage speculation even if they are unsure of the 'exact' answer.	Are the students capable of transferring the information from the day's activity to another rational function?

Questions for the observer:

- We chose to use poster paper so the entire class could view the functions; would overhead transparencies have had a better effect because we could have put the functions over top of one another and compared the graphs?

- Is a rational function too big of a jump for regular algebra I students?

- Besides hint cards, how else could we encourage students to look at the non-integer values between -2 and -4?

- Should the students have seen a function such as $y = \frac{1}{x}$ before this lesson?