Mathematics Lesson Plan for Seventh Grade

For the Lesson on Friday, October 17, 2003
At the Lansing Center, Lansing, Michigan
Instructor: Akihiko Takahashi

1. Title of the Lesson: How many isosceles triangles can you find?

2. Goal:

   a. To deepen students' understanding of the concept of isosceles triangle through open-ended problem solving
   b. To help students develop reasoning skills through developing and examining a variety of isosceles triangles by using the properties of an isosceles triangle.

3. Relationship of the Lesson in the Michigan Curriculum Framework
   Stand II. Geometry and Measurement
   Content Standards 1: Students develop special sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes. (Shape and Shape Relationships)

	Elementary		Middle School
1. Special sense relies on the ability to recognize and describe shape	Recognize and name familiar shapes in one, two, and three dimensions, such as lines, rectangles and spheres, and informally discuss the shape of a graph.		Distinguish among shapes and differentiate between examples and non-examples of shapes based on their properties; generalize about shapes of graphs and data distributions
2. Recognize attributes and characteristics of shapes is a prerequisite for understanding	Describe the attributes of familiar shapes		Generalize the characteristics of shapes and apply their generalizations to classes of shapes
4. Drawing and constructing shapes in two and three dimensions are important ways to represent the world	Draw and build familiar shapes.	This Lesson	Construct familiar shapes using coordinators, appropriate tools (including technology), sketching and drawing two- and three-dimensional shapes.
6. Figures that are alike in size and/or shape and figures that have special relationships to each other lead to important generalizations.	Recognize parallel and perpendicular line segments and figures that have similarity and/or congruence.		Generalize about the common properties of similar, congruent, parallel and perpendicular shapes and verify their generalizations informally.
7. Shape, shape properties, and shape relationships help students to describe and make sense of the physical world and to solve problems.	Use shape, shape properties and shape relationships to describe the physical world and to solve problems.		Use shape, shape properties and shape relationships to describe the physical world and to solve problems.

4. Instruction of the Lesson

NCTM Standards expect middle-grade students to explore a variety of geometric shapes and examine their characteristics by using manipulatives and dynamic geometry software in order to provide a rich context for the development of mathematical reasoning, including classifying and defining geometric objects.

Prior to the seventh grade, students learned about an isosceles triangle as a triangle with a pair of sides with equal length. Students are expected to develop the concept of basic shapes, including isosceles triangle, through hands-on activities in elementary grades. However, younger students sometimes have difficulty recognizing a shape like figure 1 as an isosceles triangle, because this does not look like their image of an isosceles triangle. Although the shape might not look like an isosceles triangle to the student, middle grade students are expected to examine such a shape by using its definition and explain their thinking.

In order to build upon students' previously learned knowledge, this lesson is designed as an introduction of a series of lessons designed to provide students with opportunities to classify and define basic geometric shapes by using their definitions. For example, students are expected to identify a shape like figure 1 by informally using its definition, a triangle with two sides of equal length. In order to do so, students need to identify two sides with equal length by using their previous learned knowledge.

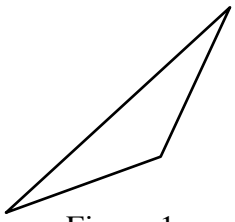
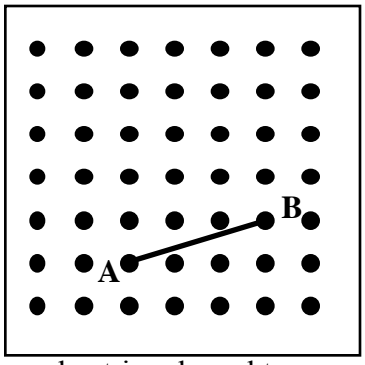


Figure 1

To provide an opportunity to extend their previous knowledge, this lesson employs the following open-ended approach through a hands-on activity.

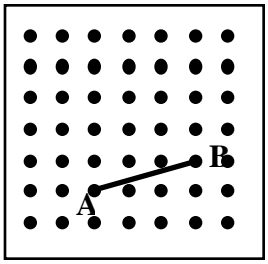
By using a line segment AB as one of the sides, make an isosceles triangle ABC on your geoboard. How many isosceles triangles can you make on your geoboard? Find as many isosceles triangles as possible.



To maxmize students opportunity to enrich their understanding of isosceles triangle and to use their informal reasoing, the line segment AB has been chosen. There are nine possible Cs, which can make ABC as an isosceles trianle on the geoboars. Since the length of AB is not obvious, students are expected to not only identify as possible isosceles triangles but also explain a reason why the triangle is a isosceles triangle. In other words, a student needs to identify which two sides are equal length and why these two sides are equal. It is expected that students will be able to enrich their concept of isosceles triangles and understand a way to explain their thought process. Since this is an important opportunity for students to develop their reasoning skills toward formal proof, the lesson will focus on providing such an opportunity through discussion in this lesson. Moreover, students are expected to fine more possible Cs by extending the size of geoboard. By using various isosceles triangles that students find, they will be given an opportunity to classify their triangles into three categories, a triangle with AB=BC, with AB=AC, and with AC=BC in order to see if they found all the possible solutions.

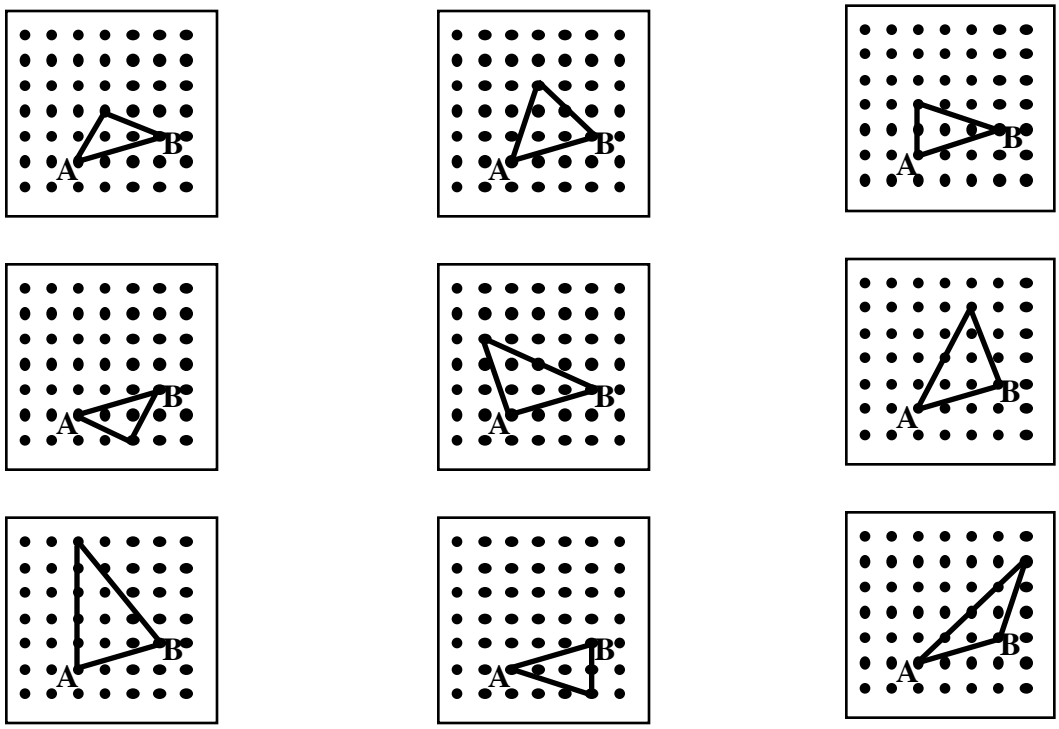
At the end of the lesson, dynamic geometry software will be used to see if all the possible Cs were found.

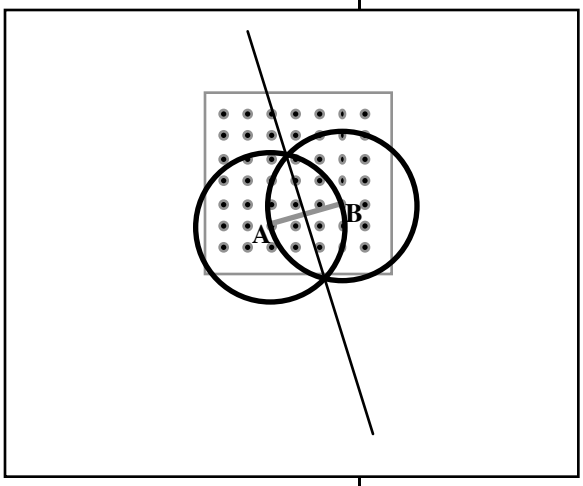
5. Lesson Procedure

Learning Activities Teacher's Questions and Expected Students' Reactions	Teacher's Support	Points of Evaluation
1. Introduction 1) Help students recall their previous knowledge about triangle, such as isosceles triangle and equilateral (regular) triangle. 2) Help students recall a use of geoboard. • Ask students to make a triangle on a geoboard by using a rubber band. • Ask students make an isosceles triangle on a geoboard by using a rubber band and explain why the triangle is an isosceles triangle. • (Ask students make an equilateral (regular) triangle on a geoboard by using a rubber band and explain why the triangle is an equilateral triangle.)	Write an informal definition of isosceles triangle on the blackboard using students' words such as a triangle with a pair of sides with equal length.	Do the students recall what an isosceles triangle is?
2. Posing the Problem 1) By showing students the geoboard, ask students to draw a line segment AB on the Geoboard by using a rubber band. 2) Pose the following problem to the students. By using a line segment AB as one of the sides, make an isosceles triangle ABC on your geoboard. How many isosceles triangles can you make on your geoboard? Find as many isosceles triangles as possible.  	Give a geoboard to each pair of students. By using an actual geoboard, show where the line segment AB should be. Pose the problem in written form on the blackboard. Give students enough worksheets so that they can draw the isosceles triangles that they find with the geoboard.	Can all the pairs of students draw the line segment AB on their geoboards? Do students understand the problem?
Students' anticipated solutions Nine isosceles triangles can be made on a geoboard by using the line segment AB as a side.	Encourage students to talk freely about their ideas for finding isosceles triangles with their partners.	

Learning Activities Teacher's Questions and Expected Students' Reactions	Teacher's Support	Points of Evaluation
3. Discussion (1) Ask students to explain their solutions. Students are expected to explain why two sides of the triangle are equal by using their previous knowledge. Their previous knowledge might include congruence of right triangles and the Pythagorean theorem. (2) Help students categorize all the isosceles triangles they find into three groups in order to see if there are no other isosceles triangles that can be made on the geoboard. • Using the blackboard to help students organize all the isosceles triangles by using students' worksheets	Give a strip of tape to help them to compare the length of sides of a triangle informally if students request it. Encourage students to explain the reason why they think their triangle is an isosceles triangle without using any other material such as a piece of tape or a pencil to directly measure the length of sides.	

AC=BC **AB=AC** **AB=BC**



Learning Activities Teacher's Questions and Expected Students' Reactions	Teacher's Support	Points of Evaluation
4. Extending the Problem If we have a larger geoboard with more pegs, can we fine more isosceles triangles by using a line segment AB as one of the sides? 1) Let students draw all the isosceles triangles that they have found on their new worksheets. 2) Encourage students to find more triangles by using the categories that they used to organize their solutions. 3) Let students show the class new triangles that they have found. 3) Help students recognize that all the Cs, which make triangle ABC isosceles triangles, are on the following geometric figures: • All the Cs that make triangle ABC with AC=BC are on the perpendicular bisector of AB • All the Cs that makes triangle ABC with AB=AC are on the circumference of a circle with the radius of the length equal to AB and A as its center. • All the Cs that makes triangle ABC with AB=BC are on the circumference of a circle with the radius of the length equal to AB and B as its center.	Give students another worksheet with more pegs.  Show the above figure to the students by using dynamic geometry software	
4. Summing up (1) Using the writing on the blackboard, review what students learned through the lesson. (2) Ask students to write a journal entry about what they learned through the lesson.		

5. Evaluation
   a. Were students able to deepen their understanding of the concept of isosceles triangle through open-ended problem solving?
      Did they find several isosceles triangles by using a geoboard?
      Did they identify two sides with equal length to examine various isosceles triangles during their problem solving as well as during the class discussion?
   b. To help students develop reasoning skills through developing and examining a variety of isosceles triangles by using the properties of an isosceles triangle:
      Were they able to explain their thinking formally and informally during the class?