3rd Grade Lesson—Fractions

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Focus Question

Does the context of liquid measurement in this lesson help students deepen their understanding of fractions?

Goals

- Students will deepen their understanding of fractions by applying them to a new context: measuring volume;
- Students will see that fractions are useful for solving problems about liquid quantity;
- Students will learn to pay attention to the number of intervals on a (measurement) scale;
- Prepare students to understand the area model of fractions which they will encounter in 4th grade;
- Students will learn the vocabulary: <u>fraction, numerator, denominator.</u>

Relation of the goals to standards

<u>Illinois Assessment Framework for third grade:</u>

6.3.03 Recognize a fraction represented with a pictorial model.

<u>Japanese Course of Study, third grade:</u>

(5) To enable children to understand decimal numbers and fractions in simple cases and appropriately use them, thereby to gradually appreciate their significance.

A. To use decimals or fractions to represent the size of fractional parts or the size of parts made by equally dividing. Furthermore, to know about the notations of dccimals and fractions.

NCTM Curriculum Focal Points for Grade 3

Number and Operations: Developing an understanding of fractions and fraction equivalence

Students develop an understanding of the meanings and uses of fractions to represent parts of a whole, parts of a set, or points or distances on a number line. They understand that the size of a fractional part is relative to the size of the whole, and they use fractions to represent numbers that are equal to, less than, or greater than 1. They solve problems that involve comparing and ordering fractions by using models, benchmark fractions, or common numerators or denominators. They understand and use models, including the number line, to identify equivalent fractions.

Unit plan:

The unit is based on the fractions unit in the translated Japanese textbook from Tokyo Shoseki (2006).

Lesson 1: Students measure their armspan with a strip of adding machine tape marked only at 1 meter; they cut it at their own armspan length. The problem is raised: how to compare the lengths of their armspans without physically matching up the tapes.

Lesson 2: Students hear the story of Susan and her friend Taylor (who lives in another state) and how they decided to compare their armspans—both of them are 1 meter and a little more; the question is how to describe the size of the "little more." Students learn how to express fractional parts of a 1-meter length, e.g 1/3, 1/2, 1/5, 1/4.

Lesson 3: Students learn to express multiples of a fractional part, e.g. 2/3 meter is two segments of 1/3 meter.

Lesson 4 (this lesson): Students learn that fractions can be used to measure volume of a liquid, e.g. 1/5 liter, and learn the vocabulary of fractions (numerator, denominator).

Lesson 5: Students connect fractional lengths to locations on a number line.

Lesson 6: Using a number line, students make a connection between tenths as fractions with tenths as decimals (prior learning), using the number line.

Lesson 7: Addition of fractions with the same denominator, with sums less than one.

Lesson 8: Addition of fractions with the same denominator, with sums less than or equal to one.

Lesson 9: Subtraction of fractions with the same denominator, minuend less than 1.

Lesson 10: Subtraction of a fraction from 1.

Considerations in planning the unit and lesson

Fractions are a troublesome but important topic; many adults will refer to fractions as their mathematical "Waterloo."

U.S. textbooks mostly use three representations of fractions: including area (e.g. pizza slices), discrete (e.g. 3/4 of a bag of four cubes are blue), and linear. (Watanabe, 2002). Of these, the area model is most common in the early grades. But students' conception of area is not typically strong. For instance, Watanabe (2002) presented 5th graders with the task of choosing the largest cookie from three shapes, each of which was exactly half of congruent squares (see figure 1). Fourteen of sixteen students selected one as the largest.

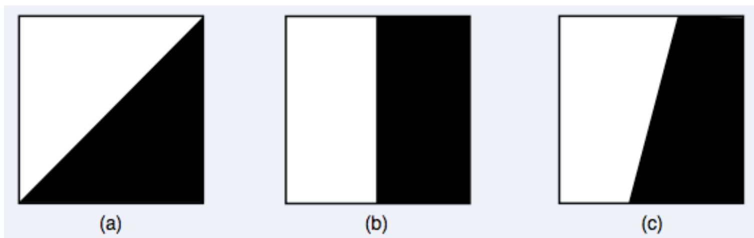


Figure 1: The cookie problem (Watanabe, 2002).

If area is not itself a stable concept when fractions are introduced, then it might make a poor choice of model for developing an understanding of fraction. The area model presents other difficulties as well; for instance, students often have trouble understanding fractions greater than one (Thompson and Saldanaha, 2003, cited in Watanabe, 2006).

The approach to fractions in Japan differs significantly from the U.S. approach (Watanabe, 2006). Fractions are typically introduced in the context of measurement; the area model is delayed until 4th grade, and when it is, rectangles, rather than circles, are used; and non-unit fractions are treated as repetitions of a unit fraction—2/3 is twice one-third—which makes it relatively easy to consider an arbitrary number of 1/3s strung together.

The fractions unit in the third-grade textbook from Tokyo Shoseki begins with length and then moves to liquid measurement, models which were used earlier in the year to introduce decimals. The length model translates easily to a number-line model; the liquid measurement model used in this lesson translates easily to an area model. Besides the change in context, students also have to work with a vertical scale, which is perhaps less concrete than a joining of segments of tape.

McDougal taught this unit to 5th graders. The first problem of the textbook lesson is to determine the fractional part shown in the figure at right. Several of the 5th-grade students had trouble determining the correct denominator. Looking at a cube filled with liquid, many students counted just the scale markings and decided that the quantity was 2/4.

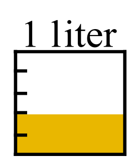


Figure 2

This or related problems crop up in many different contexts. Counting on a number line, some students will start counting on at the starting number instead of at the next number. During lessons on elapsed time using a number line representation, given a problem such as "What is 40 minutes after 9:50?" some students counted minutes ("10, 20, 30, 40") starting at 9:50 instead of 10:00. Using a ruler, students will sometimes align the mark for 1 inch at one edge of the object being measured.

The design of this and earlier lessons is aimed at teaching students to appreciate that the tick marks on a scale indicate intervals of quantity, without themselves having value.

Teaching the Vocabulary

The last part of the lesson introduces the terms "numerator" and "denominator." We think that the Japanese textbook delays this vocabulary until now (the third lesson of the unit) so that the concepts to which they refer are more firmly established and linked to more than one context (length and volume). We agree with this reasoning.

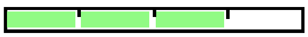
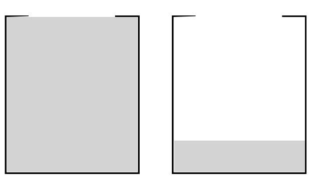
We differed as to how to introduce the new vocabulary. One member of our team felt strongly that we should teach their related words *denominate* (to name something) and *numerate* (to count), perhaps linking them to related words the students might see—denomination, nominate, nominclature, and *nom* and *nombre* (French and Spanish for *name*, respectively); or numeracy, number, enumerate. Doing so would provide another way for students to make sense of the parts of a fraction, would help them remember which one was which, and would reinforce the general message that mathematics is not arbitrary. But another member of our team felt strongly that teaching the addditional vocabulary would confuse the students, was not age-appropriate, and that the words *denominate* and *numerate* are too infrequently used to be worth teaching.

Lesson Plan

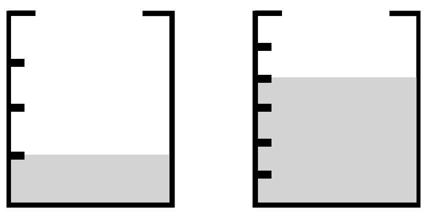
Supplies & preparation:

- Adding machine tape, cut to 1 meter, marked in 4ths (but only partway)
- Three colored strips cut to 1/4 meter, to overlay the adding machine tape
- Large pictures of Susan (with three large glasses of orange juice) and Taylor (with many small cups of orange juice)
- 1-liter cubes (x2)
- large picture showing 1.2 liters orange juice (1 cube full, one cube 1/5 full)
- large poster of diagram of 1.2 liters, w/ large 1/5 liter strips of construction paper
- diagram of 1/5 liter, one for each group, with a strip of orange construction paper glued in place to represent 1/5 liter
- additional orange strips (at least 7) congruent to the one on the diagram
- sheet of paper with the two practice problems, 1 per student.

- enlarged versions of practice problems for board.

Steps, Learning Activities, Teacher's Questions, and Expected Student Responses	Teacher Support	Points of Evaluation
Introduction		
As a brief review of previous work, show mock-up of 3/4 meter from the previous lesson.		
		
"I didn't get to see yesterday's lesson. Can someone tell me about this?"		
"I understand you helped solve the problem of comparing Susan's and Taylor's armspans. They have another comparison problem. They want to know who drinks more orange juice.		
"Susan says she drinks 5 glasses of orange juice. Taylor says she drinks 13 cups of orange juice."		
Post a picture of each. (Picture shows Susan w/ 5 large glasses & Taylor w/ 13 dixie cups.)		Do students realize that the containers need to be the same?
"Who drinks more?"		
"If we can't tell from this picture, how can we tell?"		
"We realized that we needed to use the same size container. We both have some 1-liter boxes like this, so we poured Susan's juice into it, and this is what it looked like."		
Post a photograph showing 1 full liter box and a partly-filled 1-liter box.		
"We made this diagram to make it easier to see." Post diagram:		
		
Posing problem		

"Please help me figure out how to describe this extra amount of orange juice." Pass out to groups a picture of a 1-liter box w/ a 1/5-orange strip that has been glued-down in the position of the liquid. Also give each group 7 identical strips.	Post: "Please help me figure out how to describe this extra amount of orange juice."	Do students think to use fractions to solve this problem? Do students use the strips as a measuring tool to determine fractional parts? Do students make a connection to their earlier work with fractions of a meter?
<u>Anticipated responses</u>		
• 1/5 liter (correct) • 1/4 liter (by counting only the number of additional strips that will fit)	"How do you know it is 5ths (or whatever)?"	Do students identify the unit ("liter")?
• 5 of these make 1 liter. • 1 out of 5 parts • 1 out of 4 parts • unsure how to proceed	"Didn't you solve a problem about Susan's armspan?"	
Discussion Move the strip showing 3/4 (from the introduction) off to the side, rotated vertically.		
Groups present, starting w/ 1/4 (if it came up), then 1/5, then others. Groups can manipulate large versions of their 1/5 strips on diagram on the board. Teacher transcribes student method.	"How can we decide which is right? What about this 3/4 strip?"	

Ask groups to share questions or disagreements they had.		Do students understand they need to count spaces, not lines?
Second and Third Problems		
"Would you like to try another problem like this?"		Are students enjoying this?
Ask students to think about these two new problems:  (a) (b)		
Post on the board, ask students to think silently for 2/5 minute.		
Then distribute diagrams to each group. Diagram (a) will have a colored strip glued down, but the group will only have one additional matching strip. Diagram (b) will be shaded only; the group will have to		Are there fewer difficulties now? Do students use the extra 1/4-liter strip, to determine the fractional part?

Anticipated responses:

Problem (a):

- Use the provided strip to count 4 intervals; answer = 1/4
- Use the provided strip to count 3 remaining intervals; answer = 1/3
- Use the tick marks to determine that the parts are fourths.
- Count tick marks; conclude that the parts are thirds.

Anticipated responses for problem (b) are similar to problem (a), without a provided strip.

Do students make the transition to using the scale on (b) to determine the fractional part?

Do students think about counting strips (intervals) or do they count the tick marks?

Discussion

Invite one group to present each problem.

If the students refer to the spaces in between the tick marks in problem (b), introduce the term “interval”.

Do students remember “interval” from the earlier lessons on elapsed time?

Introduce vocabulary

Ask students to take out their math notebooks and turn to the vocabulary section.

Explain that numbers like 1/5, 1/4, and 4/6 are fractions. Write on the board:

“1/5, 1/4, 4/6 are fractions”

Tell students to write this in their vocabulary.

Point out that "4" has different meaning in 1/4 than in 4/6. So we need special names for the parts of a fraction.	Write 1/4 and 4/6 and circle the 4s. Write "different meaning."	
"What might you call this number?" Point to the denominator. "What might you call this number?" Point to the numerator. Teacher writes student ideas.		Does the opportunity to make up their own terms make it easier to learn the conventional terms?
<u>Anticipated responses:</u> • derivations based on physical placement (e.g "top" or "upper" and "bottom" or "lower") • derivations based on meaning (e.g. "counter" and "size")	"What's different about 4 in 1/4 vs. 4/6?"	
"In your notebooks, please copy this picture of 4/6." "Now please copy this as I write: 4 — <u>numerator</u>: counts how many 6 — <u>denominator</u>: the name of the fractional part		
Practice "Would it be helpful to practice a little with this vocabulary?" Offer the following problems: Which is the denominator and numerator in each of these fractions? 1/4, 5/8, 7/9 Please write a fraction that has 7 as the denominator and 3 as the numerator. Each student receives these on a sheet of paper.		Do students acknowledge the need for practice? Does this promote a positive attitude towards the task?
Discussion		

Conclusion & Assessment		
Remind students that we started out with the problem of describing how much orange juice Susan drinks so we can compare it with how much Taylor drinks. Ask them to help compose an email message to Taylor and her parents describing what we found out and how, so that they will know what to do.	Post: "Please compose an email message to Taylor and her parents describing what we found out and how, so that they will know what to do."	Can students explain how they determined the size of the fractional part?

Blackboard Plan

Bibliography

Thompson, Patrick W. and Luis A. Saldanha, (2003). "Fractions and Multiplicative Reasoning." In *A Research Companion to* Principles and Standards for School Mathematics, Jeremey Kilpatrick, W. Gary Martin, and Deborah Schifter, eds., pp. 95-113. National Council of Teachers of Mathematics, Reston, Va.

Watanabe, Tad (2002). "Representations in Teaching and Learning Fractions." *Teaching Children Mathematics,* 8 (April), 457-463.

Watanabe, Tad (2006). "The Teacheng and Learning of Fractions: A Japanase Perspective." *Teaching Children Mathematics,* 12 (March), 368-374.