6th Grade Mathematics Lesson Plan

For the lesson on May 13, 2006
Chicago Lesson Study Conference 2006
6th grade students from the Oscar Mayer School
Instructor: Akihiko Takahashi

1. Title of the lesson: Does the area of the quadrilateral change?
2. Goal:
   - i. To deepen students' understanding of the concept of area through problem solving
   - ii. To develop the formula for finding the area of parallelogram
   - iii. To help students become good problem solvers by
     - i. encouraging students to use their prior knowledge to examine a problem situation to develop their ability to use logical reasoning to make conjectures, and
     - ii. encouraging students to examine and justify solutions presented by their peers in order to find a solution to the problem.
   - iv. To provide opportunities for students to recognize the importance of working with their peers in order to deepening their understanding of mathematics
3. Relationship between mathematical contents of this lesson and the Principles and Standards for School Mathematics (NCTM 2000).

<u>Geometry Standard for Grades 3–5</u>

- Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes.
- Describe location and movement using common language and geometric vocabulary.
- Use geometric models to solve problems in other areas of mathematics, such as number and measurement.

<u>Measurement Standard for Grades 3–5</u>

- Understand the need for measuring with standard units and become familiar with standard units in the customary and metric systems.
- Explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way.
- Develop, understand, and use formulas to find the area of rectangles and related triangles and parallelograms

<u>Measurement Standard for Grades 6–8</u>

Develop and use formulas to determine the area of triangles, parallelograms, and trapezoids.

This Lesson

<u>Measurement Standard for Grades 6–8</u>

Develop and use formulas to determine the circumference of circles and the area of circles and develop strategies to find the area of more-complex shapes.

4. Instruction of the Lessons

According to the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000), one of the major goals of measurement in the 6-8 grade band is to develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more-complex shapes. The standards expect students to develop the formulas by using students' previous learning, and use these formulas not only for finding the area of shapes but also for developing formulas for finding the area of other shapes.

One of the challenges for students is to understand the concept of measuring the area of the basic shapes and use them to develop the formula for other shapes. Although students in fourth grade learn the basic ideas for measuring the area of rectangles and squares by tiling the unit squares in the shape and find the area of the shape, they often struggle using this idea to develop the formulas for finding the area of parallelograms, triangles, trapezoids, and rhombi. For example, I often observe students writing the following equations in order to find the area of the irregular shape (Figure 1).

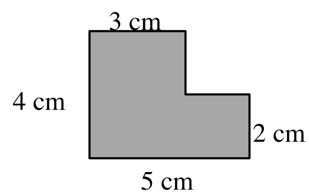


Figure 1.

$4 \times 5 \times 2 \times 3 = 120$ or 4+5+2+3=25

These students are typically able to find the area of the shape by counting the number of the squares when they draw unit squares in the shape. For these students, the relationship between counting the number of unit squares and the use of the formulas for finding the area might not be clear. The similar lack of understanding can be observed even after the students are introduced to the formula for finding the area of parallelograms. One example is that students often use the measurements of the lengths of the sides to find the area in the problem like Figure 2. These students with the following equation to find the area.

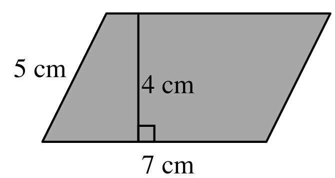


Figure 2.

$5 \times 7 = 35$

Another example, which I observed a couple of weeks ago, is that some of the students in the class cannot identify the base of the parallelogram (Figure 3), although many of them recognize the height of the parallelogram is 2. They have been told that base and height are two important words that describe parallelograms. One student argues that the base should be seven because you need to add one to the measurement of the longer side of the parallelogram from another side, otherwise you cannot see the actual measurement of the base. This example demonstrates that it might not be easy for some students to identify base and height of parallelograms even though the term base and height are introduced.

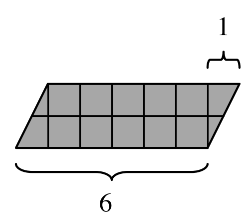


Figure 3.

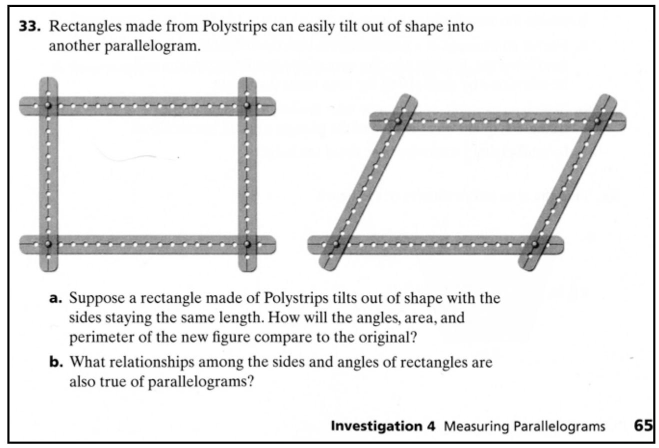
In order to help students understand the idea behind the formulas, it is important for students to revisit the meaning of multiplication and help students find the relationship with the concept of measuring the area. For example, students must understand why we can use multiplication to find the number of the unit squares in a shape.

In particular, we want students to understand why we need the measurement of the height and base of the parallelogram but not the measurements of two adjacent sides.

The problem for this lesson is based on the situation where students often reach a wrong estimation based on their lack of deeper understanding of measuring the area a parallelogram.

Students typically learn the formula for finding the area of rectangles and the formula for finding the area of parallelograms separately. In many cases they learn each formula in different grades. Although many textbooks try to help students see the relationships between the formulas for finding the area of rectangles and parallelograms, it is sometimes not clear for many students why the area of rectangles can be found by multiplying two adjacent sides but not for parallelograms. To provide students opportunity to realize the relationship between these two formulas, we often ask students to compare the area of a rectangle and a parallelogram with the same sizes of adjacent sides.

The new version of the Connected Mathematics series, Connected Mathematics 2 (CM2), includes this problem as a part of the connections in the Covering and Surrounding on page 65. This problem cannot be found in the previous Connected Mathematics series. The CM2 includes this problem, Figure 4, at the end of the Investigation 4 where students have already learned how to find the area of parallelograms. The problem is designed for students to see the connection between the areas of the rectangles and parallelograms with the same perimeters. While CM2 uses this problem after students to learn how to find the area of rectangles, there is another way to use this problem. In fact, Japanese teachers often use this problem as an introduction to learn how to find the area of parallelogram. By showing this situation, a teacher provides students an opportunity to realize that multiplying two adjacent sides may not be appropriate to find the area of the parallelograms. Students are asked where they need to measure in order to find the area of the parallelograms by using a formula similar to the ones for finding the area of rectangles and squares. Students are expected to find that the base and the height are two important measurements to find the area of parallelograms by changing the shape of the parallelogram into rectangle without changing its area. On the other hand, the Connected Mathematics Series, both previous version and CM2, simply give students the base and the height as the two words to describe parallelograms.


33. Rectangles made from Polystrips can easily tilt out of shape into another parallelogram.

a. Suppose a rectangle made of Polystrips tilts out of shape with the sides staying the same length. How will the angles, area, and perimeter of the new figure compare to the original?

b. What relationships among the sides and angles of rectangles are also true of parallelograms?

Investigation 4 Measuring Parallelograms **65**

Figure 4. The problem in the CM2

Thus, one goal for this lesson is to help students recognize that the base and height are the crucial measurements required to find the area the area of a parallelogram. To accomplish this goal, this problem will be presented to students before the terms base and height are introduced, which is different from the way the problem is used in CM2.

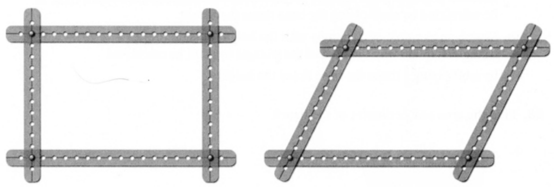
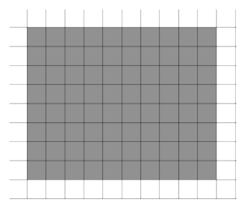
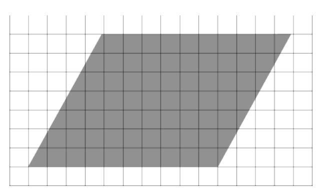
In order for students to focus on recognizing the base and the height of a parallelogram and develop the formula for finding the area of parallelograms, the original problem from the CM2 (Figure 3) will be modified for this lesson in the following ways:

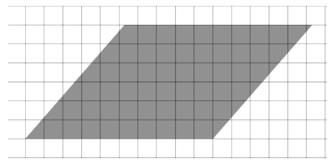
i. The question for the problem will be focus on the change of the area and exclude the question b, asking the relationship among the sides and angles of the rectangle and the parallelograms.

ii. In order for students to easily recognize the change of the areas among the rectangle and the parallelograms, the dimensions of the rectangle made from Polystrips, 8 cm wide and 8cm long, will be given to the students.

iii. To help students visualize the movements of the shape and recognize the change of the area, the teacher will use the enlarged sized of the shape made from the Polystrips for the blackboard and Geometer Sketchpad for the screen.

5. Flow of the Lesson

Learning Activities Teacher's Questions and Expected Students' Reactions	Teacher's Support	Points of Evaluation
1. Introduction to the Problem By watching the teacher's demonstration students recognize that a rectangle made from Polystrips can easily tilt out of shape into another parallelogram.  **Posing the problem** What is the area of the rectangle?  Anticipated students' responses a. Using the formula for finding the area of a rectangle, $length \times height$. b. Counting the number of unit squares one by one When the shape made from Polystrip is changed from the rectangle to a parallelogram, does the area of the shape change or stay the same? Anticipated students' responses a. The area will be different if the shape changed from the rectangle to a parallelogram. b. The area will not change because the sizes of the Polystrips stay the same even though the shape becomes slanted. c. The area of the shape will stay the same until a certain point but will change after that point	Using a model made by the Polystrip on the blackboard the teacher will change the original rectangle to show several different size of parallelograms. To help students recognize the relationship, the size of the model rectangle will be larger than the rectangle of the problem (base 10 unit length, height 8 unit length). Because the students have not learned the formula for finding the area of parallelogram, the shapes will be presented with a grid paper in order to help students see the number of unit squares.	Do students understand the situation? Do students recall a couple of ways to find the area of rectangles and parallelograms?
2. Problem Solving Working as a group, students try to find the answer to the problem. 1) Find the area of a parallelogram with the height of 7 square units to see if the area of the parallelogram will be different from the original rectangle.  a. Cut and rearrange a piece of the parallelogram to change the shape into the rectangle and find the area by using the formula for finding the area of a rectangle, $length \times height$. b. Cut and rearrange a piece of the parallelogram to change the shape into the rectangle and count the number of unit squares one by one 2) Find the area of a parallelogram with the height of 6 square units to see if the area of the parallelogram will be different from the	Encourage students to use the knowledge that they learned previously to find the areas. Provide students with worksheets to keep their work for the whole class discussion. Each group may use a large sheet of paper with the shape of the parallelogram to prepare for presenting the solution. Each group may use a	Can each group of students find a way to find the areas by using their previous learning?

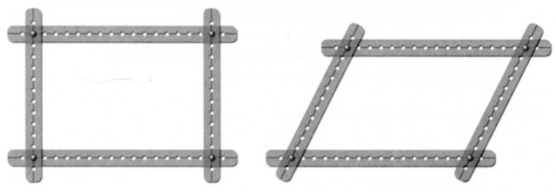
previous shapes.  a. Cut and rearrange a piece of the parallelogram to change the shape into the rectangle and find the area by using the formula for finding the area of a rectangle, $length \times height$. b. Cut and rearrange a piece of the parallelogram to change the shape into the rectangle and count the number of unit squares one by one	large sheet of paper with the shape of the parallelogram to prepare for presenting the solution.	
3. Class discussion (1) Ask students to explain their solutions to the other students in the class. (2) Facilitate students' discussion about their solutions, then lead students to realize that multiplication can be used for finding the area of the parallelogram. (3) Help students to recognize that the area of the shape made by the Polystrips keeps changing when the shape is slanted but its perimeter stays the same. (4) Introduce the terms, base and height, as important measurements to find the area of a parallelogram. Help students to understanding why the area of parallelograms can be found by multiplying base and height.	Write student's solutions and ideas on the board in order to help students understand the discussion. To help students visualize the movements of the shape and recognize the change of the area, the teacher will use Geometer Sketchpad for the screen	Can students explain their solutions to their peers? Can students examine and justify solutions presented by their peers?
4. Summing up (1) Using the writing on the blackboard, review what students learned through the lesson. (2) Ask students to write a journal entry about what they learned through this lesson.		Can students articulate the importance of measuring the base and the height to find the area of the parallelograms?

References

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, Virginia: National Council of Teachers of Mathematics.

Lappan, G et al. (2004). *Covering and Surrounding, Connected Mathematics 2*. Boston, Massachusetts: Parentice Hall.

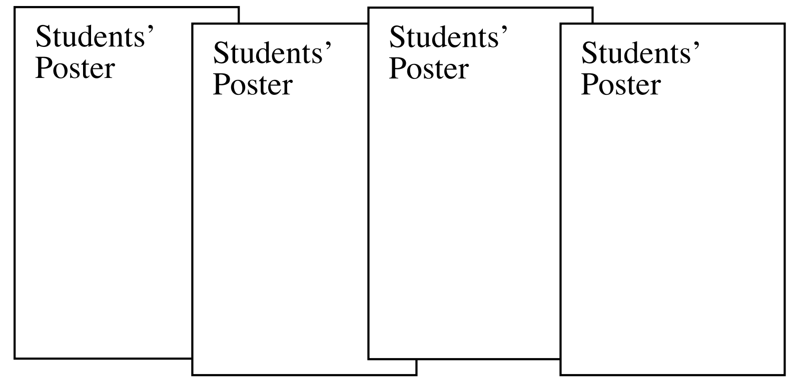
Plan for Board Work

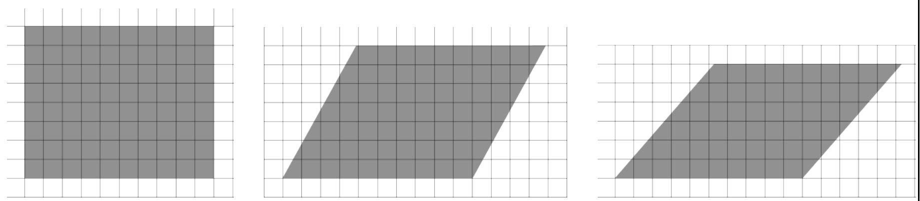


a rectangle to parallelograms

Does the area of the quadrilateral (four sided shape) change?

- The area will be different because the parallelogram will be shorter than the rectangle.
- The area will not change because the sizes of the Polystrips stay the same even though the shape becomes slanted.





$10 \times 8 = 80$ | $10 \times 7 = 70$ | $10 \times 6 = 60$

The area will be different if the shape that made from Polystrips changed from the rectangle to a parallelogram because the height of the parallelogram will be shorter than the rectangle.

- The height of a parallelogram is the perpendicular distance between the two parallel sides, the top and the base.

The base and the height are the two important measurements to find the area of parallelograms.

The area of parallelograms can be found by using the formula

$base \times height$