6th Grade Research Lesson

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Title: Art or Junk? Discovering the Triangle Inequality

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Focus questions

Can students discover the triangle inequality for themselves and articulate why it is true? And, for those students who do not discover it themselves, can they learn it when their peers articulate it?

Goals of the lesson

- Students will learn that some combinations of three segments cannot be attached at the ends to make a triangle, will discover the triangle inequality (i.e. the length of the longest side of a triangle must be less than the sum of the lengths of the other two sides), and will understand why it is true;
- Students will see that mathematics is useful for guiding real-world decisions;
- Students will gain an appreciation of the value of organizing data in order to make patterns easier to discern.

Relationship to standards

Illinois State Goal 9A (all grades): "Demonstrate and apply geometric concepts involving points, lines, planes, and space."

Here are the relevant Performance Descriptors:

Stage E (5th grade)		Stage H (8th grade)
1. Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes.	this lesson →	4. Determine if a triangle is possible using side lengths and the triangle inequality.

Unit Plan

Lesson 1: Students see that triangles are used in art (sculpture) and construction (bridges, various buildings, including the John Hancock tower). Students build as many different triangles as they can using sides of length 7, 10, and 13 units and begin to understand: (a) all three angles of an equilateral triangle are congruent; and (b) using three pre-determined lengths, you always get the same triangle (SSS congruence).

Lesson 2 (this lesson): Art or Junk?—Students will discover the triangle inequality.

Lesson 3: Building quadrilaterals—Students will discover the corollary of the triangle inequality for quadrilaterals. They will also discover that quadrilaterals are *not* rigid.

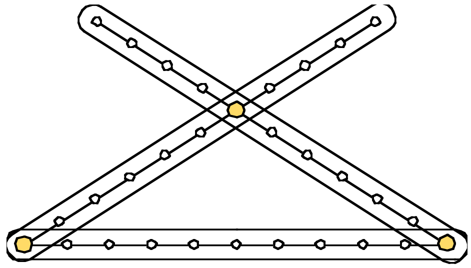
Lesson 4: Building parallelograms

Considerations in planning the lesson

The triangle inequality is frequently approached through discovery, and it certainly seems to us that students should be able to discover it. In past lessons, however, students have a lot of trouble recognizing the patterns in their data that would lead them to discover the triangle inequality, and, when they are led to see the pattern, they have a lot of trouble articulating it.

We began our research by reviewing the lesson on the triangle inequality in one of the sixth grade units of *Connected Mathematics, Shapes and Designs*.[1] The lesson as presented in *Connected Mathematics* (*CM*) has three goals: students will discover the triangle inequality; students will discover the side-side-side property of triangle congruence; and students will notice that certain side lengths yield special triangles—isosceles, equilateral, right—although these terms are introduced later. *CM* assumes that students will use "polystrips"—translucent plastic strips with equally-spaced holes either 10 or 20 units long—fastened together with brass fasteners at the vertices, to make triangles (see figure, below). The lesson suggests that students either choose their own lengths or roll three number cubes to generate lengths.

1. Lappan, Glenda, et al. *Shapes and Designs.* From *Connected Mathematics* (1st ed.). Glenview: Prentice Hall, 2002.



Polystrips attached with brass fasteners.

Past Difficulties

In the past, students have had various difficulties with this lesson as presented in *CM*. Students count holes on the polystrips instead of spaces to determine the lengths of the sides.[2] Students are dependent on the teacher to tell them what to do—e.g. they build one triangle and stop, or have difficulty generating combinations of lengths to try. They come up with very few combinations that do not form a triangle, which makes it harder for them to generalize. Students get lost in the process of making triangles or tracing them and lose sight of the central question. The polystrips, being flexible, can be attached even using lengths that should not form triangles, e.g. with lengths 10, 10, 20 (the resulting object is not flat, of course). And, technically, the shapes students make are mostly *not* triangles, because they are not simple closed polygons.

There is also a problem of motivation. How can we cause students to see the triangle inequality as useful or at least interesting? Students start out believing that any three lengths *do* form a triangle. To discover the triangle inequality, it is important for students to see enough "non-triangle" combinations that they can form a generalization. The method suggested by the text—rolling three number cubes and using their sum as a length of one side—rarely generates such combinations.[3]

Choosing the Right Learning Tools

It should be clear that polystrips have several drawbacks as a tool for this lesson, and so we searched for alternatives: connecting cubes, Cuisinaire rods, pre-cut strips of card stock, straws, and computer software (Sketchpad or Cabri). We settled on pre-cut strips of card stock. They are cheap and readily available up to 11" lengths; they don't roll around; they can be glued onto paper circles and posted onto the blackboard; and there isn't room to "fudge" whether they are long enough to make a triangle.

2. This error reflects a fundamental misunderstanding of measurement that is also discussed in the 3rd grade research lesson for this conference.

3. Using this method, the probability of generating three lengths that do not form a triangle is about 0.03.

Lesson Plan

Steps, Learning Activities, Teacher's Questions, and Expected Student Responses	Points of Evaluation
Introduction	
Review the list of true/false statements from the advance organizer students completed in Lesson 1. "Which of these statements did we answer in the previous lesson?"	Do students seem to have learned: • the order you attach segments doesn't matter (SSS congr.)? and • a triangle is simple and closed
Remind students of the triangular sculptures that they saw in the previous lesson. "Please read along in your packet on p. 4 as I tell the following story." Tell the story of the sculptor "Michaelangelo di Cardano" who decided to build a very large sculpture using steel I-beams [post drawing of an I-beam]. "He went around to different junkyards and found three I-beams: 6 ft., 7 ft., and 20 ft. They cost him $300 total. Back home, he laid them on the floor of his shop so that he could attach them at the ends." "What you you suppose was the shape of his triangle? In you packet, please circle the picture that you think is closest to what he got."	
Explain that we can't bring in I-beams, but we can use strips 6, 7, and 20 units to build a miniature version of di Cardano's sculpture, so we can see what it looks like. Pass out strips. After most students have had a chance, invite a student to use the large strips on the board to show what happens.	Do students realize that the three lengths cannot be attached at the ends to make a triangle?
"So now you know what happened: di Cardano found he could not attach his I-beams at the ends! He was able to sell the I-beams back to the junkyard, but they only gave him $100 for the scrap metal, which means he was out $200.	

Revisit the “Advance Organizer”; ask students if we have learned anything about another of the statements in it.

Ask students to write what they learned on p. 4 in their packet.

Do students realize that #2 is false (“Given any three sticks, you can always attach them at the ends to make a triangle.”)

“But this leads to another question [p. 5 in the student packet]. If di Cardano knew more about triangles, could he have predicted this would happen and saved himself $300? or was he just unlucky?”

“Please consider the next statement in your packet and circle true or false, based on what you think.”

Post: *“If you know enough about triangles, it is possible to predict in advance whether three lengths can be attached to make a triangle.”*

Do students believe there is something to learn?

First Problem

Hand out packets.

Ask groups to inspect the packets. Each packet should include:

- circles (3 small, 3 large)
- $700 in “triangular dollars”
- 1 glue stick

Introduce the simulation: (student packet, p. 6)

“We are going to play a little game as a way to find out whether it is possible to predict whether three lengths can make a triangle.

“Imagine that you are sculptors like di Cardano. Triangular sculptures from I-beams are ‘hot’ these days, so you know that if you make them you can sell them as art for $500.

“You have called around and found three junkyards that have I-beams for sale. The junkyards will deliver the I-beams, but they are far away, and so you have to decide just from the lengths whether you will buy them.

“Page 6 in your packet lists what’s available; here are the choices.”

Post: Junkyard A: 3 ft., 5 ft., 10 ft.
Junkyard B: 13 ft., 5 ft, 6 ft.
Junkyard C: 20 ft., 20 ft., 6 ft.

"They are all the same price, $300. If you buy three I-beams and you can't make a triangle out of them, the junkyard will buy them back from you as junk, but only for $100."	
Post prices for I-beams, art, and junk.	
Remind students that this is only a game, and so if they buy a set of "I-beams" that won't work, that's ok—that could even be good—because our main goal is to learn mathematics.	Do students understand the game?
"For 1 minute, discuss with you group which one you want to buy, then I'll come around to sell you the strips."	Do students enjoy it? Are they motivated to avoid "junk"?
<u>Anticipated student responses:</u> • Students randomly pick one. • (A) because none of the numbers is too big. • (A) because the numbers are closest together. • (B) because (A) has a number that's too small, and (C) has numbers that are too far apart. • (C) because two of the numbers are the same. • (C) because in (A) and (B) two of the lengths are too short. <u>Discussion</u> Note: If an option is not selected by any group, it will stay on the board as an option for future rounds.	
Ask groups that picked (A) to come up and show their results. Ask the rest of the class, "How much should we pay them for this?" Attach the card (with the numbers) to the results from one of the groups (the glued strips), and place it on the board in the "Junk" section. Pay the groups $100.	Do students recognize this as junk? Does the "payment" help them stay engaged?
Repeat for (B) and (C). (C) will be "Art."	

Ask groups that chose (C) if they got lucky or if they saw something. Ask the class if anyone has an idea that could help us make better choices. Transcribe ideas onto poster.

Are students thinking about how to predict?

Second Problem

Same procedure as first problem, but with these choices:

Junkyard D: 6 ft., 5 ft., 10 ft.
Junkyard E: 13 ft., 6 ft, 6 ft.
Junkyard F: 20 ft., 14 ft., 6 ft.

Do students recognize that only one number in each triple has changed from before?

Refer students to p. 7 in their packet, where these are listed.

Suggest that students think about the ideas that have been suggested so far.

Are students' choices more reasoned than before?

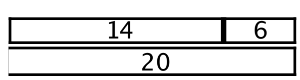
<u>Anticipated student responses</u>

- Choose by guessing.
- (D) because the shorter ones are long enough to reach.
- (D) because the numbers are closest together.
- (E) because two of them are the same (like (C)).
- (F) because two of them are long.

<u>Discussion</u>

Similar to previous discussion. For (F), show enlarged versions of these strips. Put the 14 and the 6 in a straight line: "How long are these when you put them together?" Then put the 20 beneath, like this:

Do students recognize that (F) cannot make a triangle?



"If the 6 were bigger, could we have a triangle? Would 7 be big enough? What if the 14 were bigger? What if the 20 were bigger?"

If previous ideas have referred to sides as being “too long” or “too short,” suggest that maybe this helps us understand what “too long/short” means.

Again, solicit and transcribe ideas onto the “ideas” poster.

“Maybe you can use some of these ideas on the next set of options.”

Third Problem

Post these choices:

Does the diagram of 14-6-20 help students understand that none of these will work?

Junkyard D: 5 ft., 5 ft., 10 ft.
Junkyard E: 13 ft., 7 ft, 6 ft.
Junkyard F: 20 ft., 12 ft., 6 ft.

Refer students to p. 8 in their packet.

Anticipated student responses

- Reject all options as impossible
- Reject (F) only because it didn’t work with 14, and with 12 it will be even worse.

Other anticipated responses will be similar to those from previous rounds.

Discussion

Focus the discussion now on why groups rejected each choice.

Are students clear that when a+b=c, the lengths will not make a triangle?

“Did anyone decide that (D) was definitely not going to work? Why?” Repeat for the other two. If any group tried any of these, ask them to show their result.

Return to the last true or false question (p. 5 in packet).

Do students understand this is true?

Assessment Activity

Pose some examples, ask students to predict yes or no: {40, 10, 20}, {30, 40, 20}, {20, 20, 40}, and {62, 110, 35}. Push students to explain their predictions.

Are students predicting correctly?

"How do you know if the sides are long enough to reach? What do you do with the numbers?" **Conclusion** Read the letter on p. 9 of the student packet. "Please write a letter to di Cardano. Tell him which choice he should buy now and why, and explain how he can decide in the future no matter what the numbers are."	Can they articulate their method?

Blackboard plan