


Curriculum Exploration (6b) Double Number Line Diagram (Grades 3 – 6)

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Double Number Line in CCSS


6.RR.3

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, **double number line diagrams**, or equations.




Why double number line in 3–5?

- ▶ If we want students to be able to use double number line to solve problems involving complex mathematical ideas like ratios and proportions, then students must be familiar with the tool before they reach Grade 6.
- ▶ Double number line diagram is a useful tool even for making sense of elementary school mathematics, particularly multiplication and division with fractions and decimal numbers.

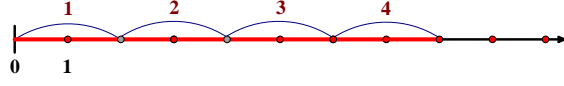



Multiplication is two-dimensional

$3 + 4 = 7$

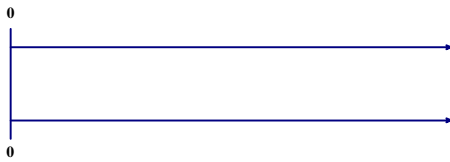


$4 \times 2 = 8$

What is a double number line?

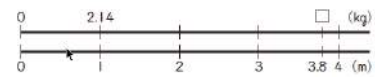
- ▶ A double number line is a visual representation of multiplicative relationship between two quantities.



Example

- ▶ Math International Gr. 5, p. A34

2 There is a 1 m pipe that weighs 2.14 kg.
How much will 3.8 m of this pipe weigh?



Another Example

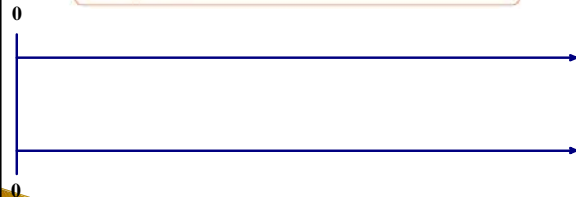
- ▶ Math International Gr. 5, p. A48

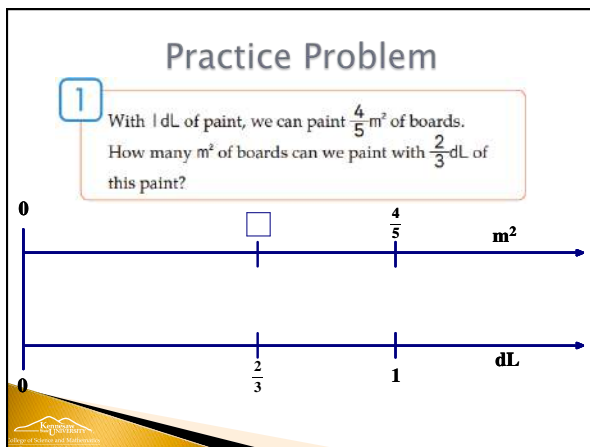
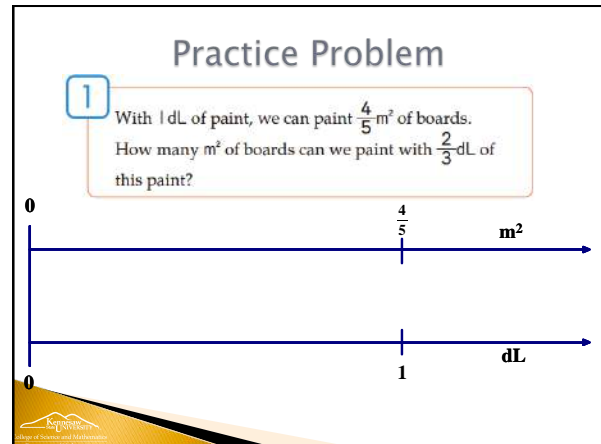
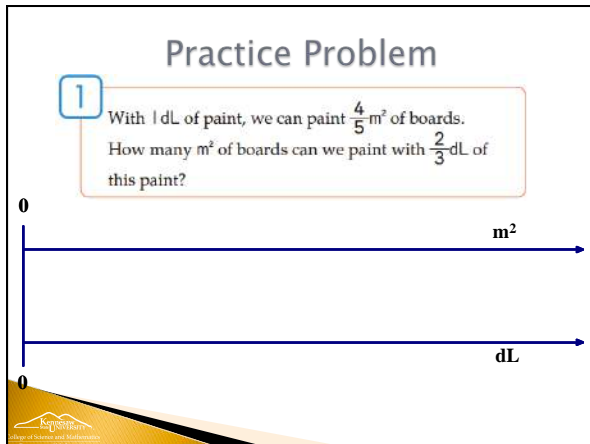
2 There is a 6.3 m iron pipe that weighs 7.56 kg.
How much will 1 m of this iron pipe weigh?



Practice Problem

1 With 1 dL of paint, we can paint $\frac{4}{5}$ m² of boards.
How many m² of boards can we paint with $\frac{2}{3}$ dL of this paint?





Two Types of Visual Model

- ▶ Models that represent the details of mathematical procedures.
- ▶ Models that represent the relationships of quantities → reasoning tools to decide what to do to solve problems.

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- Models that represent the details of mathematical procedures.

33×24

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- Models that represent the details of mathematical procedures.

33×24

$$\begin{array}{r} 33 \\ \times 24 \\ \hline 132 \\ 66 \\ \hline 792 \end{array}$$

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Two Types of Visual Model

- Models that represent the details of mathematical procedures.
- Models that represent the relationships of quantities \rightarrow reasoning tools to decide what to do to solve problems.

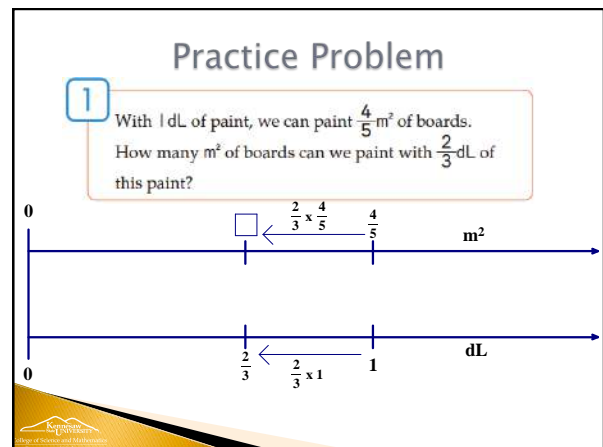
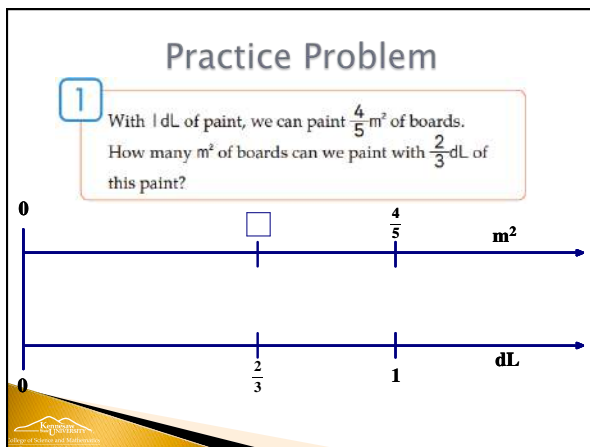
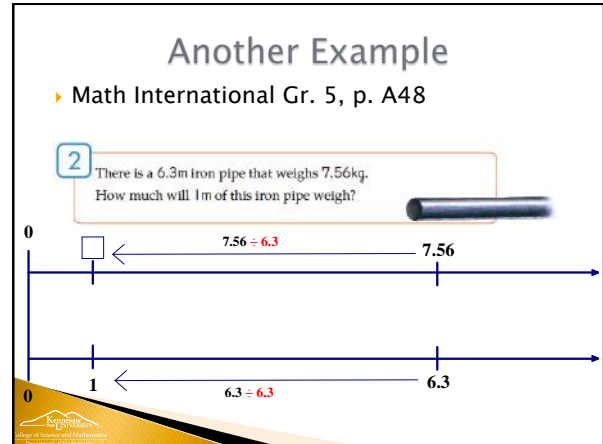
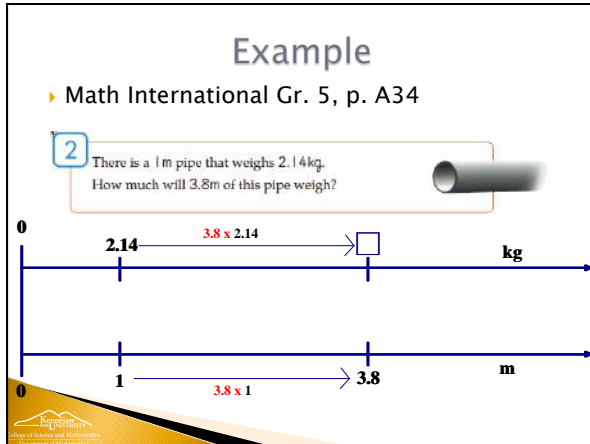
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Example

- Math International Gr. 5, p. A34

2 There is a 1 m pipe that weighs 2.14 kg.
How much will 3.8 m of this pipe weigh?

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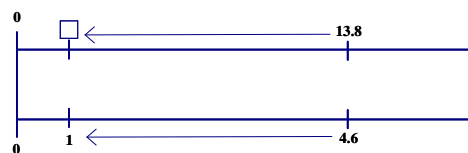


Key Properties of \times/\div

- ▶ $n \times 1 = n$: 1 multiplied by any number is the number itself.
- ▶ $n \div n = 1$: Any number divided by itself is 1.



What calculation is needed?

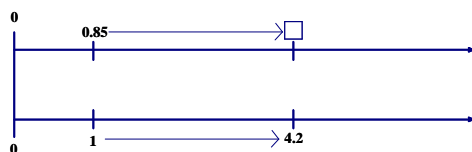


$$1 = 4.6 \div 4.6$$

$$\rightarrow ? = 13.8 \div 4.6$$



What calculation is needed?

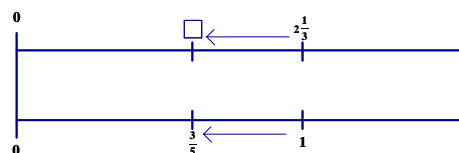


$$4.2 = 4.2 \times 1$$

$$\rightarrow ? = 4.2 \times 0.85$$



What calculation is needed?



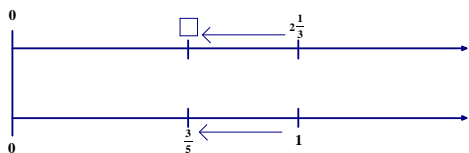
$$\frac{3}{5} = \frac{3}{5} \times 1$$

$$\rightarrow ? = \frac{3}{5} \times \frac{2}{3}$$



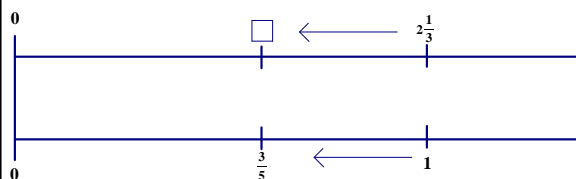
Common Misconceptions

- ▶ Multiplication makes bigger and division makes smaller.
- ▶ Students can tell that the unknown (answer) should be less than $2\frac{1}{3}$, thus they believe division is the operation needed.



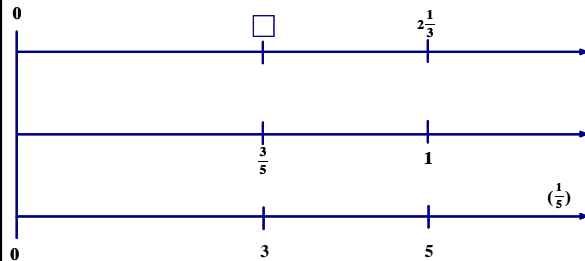
Reason about "how to calculate"

▶ $\frac{3}{5} \times 2\frac{1}{3} = ?$



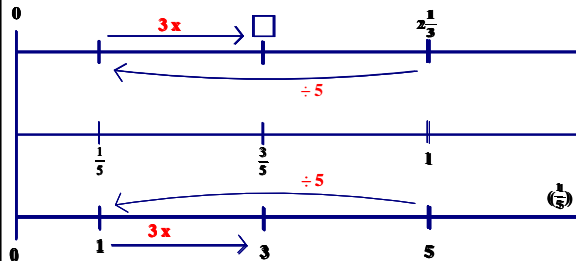
Reason about "how to calculate"

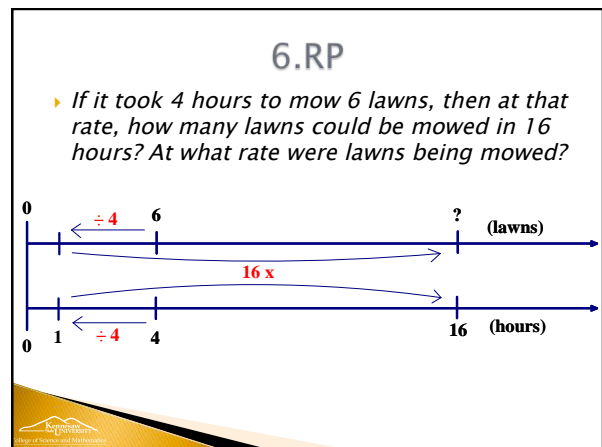
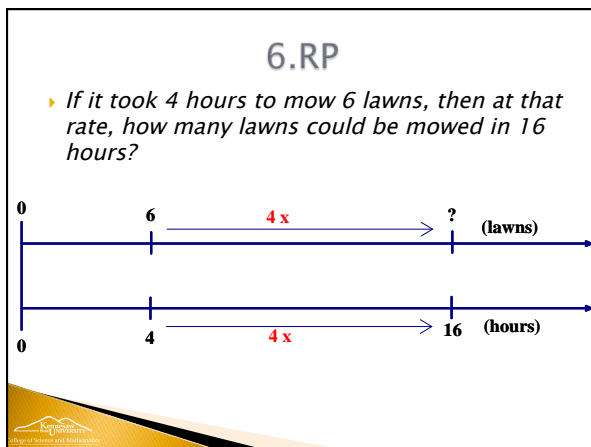
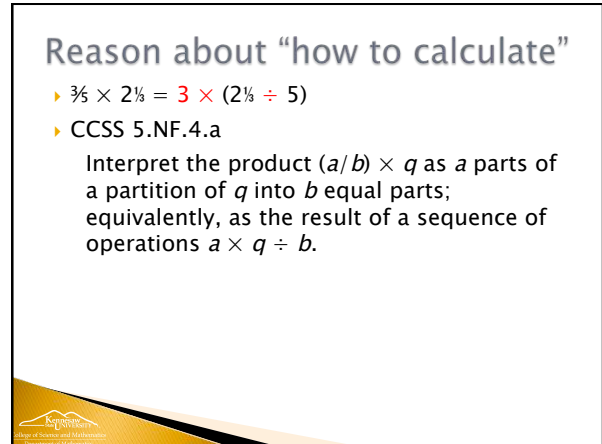
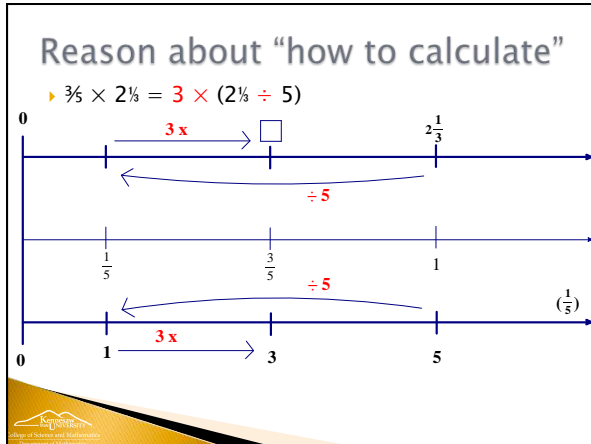
▶ $\frac{3}{5} \times 2\frac{1}{3} = ?$



Reason about "how to calculate"

▶ $\frac{3}{5} \times 2\frac{1}{3} = ?$





Developing Double Number Line

- Starting with a combination of a tape and a number line (Gr. 3 p. A91)

9 Let's Think about Multiplication

How much will 3 sheets of construction paper cost if each sheet costs 23 yen?

How much will 2 sheets of construction paper cost if each sheet costs 20 yen?

Multiplication by 10 and 100

We will buy 3 sheets of construction paper for 20 yen each. How much will it cost?

We will buy 30 sheets of construction paper for 20 yen each. How much will it cost?

Let's think about how to calculate.

Representing Problem Situations

1 We are buying 3 sheets of construction paper that cost 23 yen each. How much will it cost?

1 He costs buying 3 m of ribbon that costs 312 yen for 1 meter. How much will the price be?

Price for 1 sheet \times Number of sheets = Total price

Price for 1 m \times Length of ribbon = Total price

Let's think about how to calculate.

Gr. 3 p. A93

Gr. 3 p. A99

Grade 4 Division (Whole Numbers)

3 Let's Think about How to Calculate Division

72 sheets of colored paper will be shared equally among 3 people. How many sheets of paper will each person get?

256 sheets of colored paper will be shared equally among 4 people. How many sheets of paper will each person get, and how many will be left?

Let's think about how to calculate.

p. A33

p. A41

\times and \div as Scaling

1 An adult whale is 15 m long, and its calf is 3 m long. How many times as long is the adult whale as the calf?

2 A young giraffe is 180 cm tall, and the adult giraffe is 3 times as tall. How tall is the adult giraffe in cm?

Think about it using the diagram below.

Adult Whale: 15 m, Calf: 3 m

Adult Giraffe: 180 cm, Young Giraffe: 180 cm

Let's think about how to calculate.

Gr. 4 p. A44

Gr. 4 p. A45

× and ÷ with Decimals

15 Let's Think about Multiplying and Dividing Decimal Numbers

1 Multiplying Decimal Numbers

We bought 0.2L of juice. How much juice is there altogether?

0 0.2 0.4 (L)

What's the answer? Should we write?

Let's think about how to calculate.

2 Dividing Decimal Numbers

We bought 3L of water. If we share this water equally among 3 people, how much water will each person get?

0 1 2 3 (L)

Let's think about how to calculate.

Multiplying by Decimals

3 Let's Think about Multiplication of Decimal Numbers

1 Multiplication of Decimal Numbers

1 meter of ribbon costs 80 yen. I bought 2.5m of the ribbon. How much was the cost?

0 1 2 3 (m)

Let's think about what each number we should write.

Let's think about how to calculate.

2

There is a 1m pipe that weighs 2.14kg. How much will 2.8m of this pipe weigh?

0 2.14 4 (kg)

Reasoning with different unit

1 1 meter of ribbon costs 80 yen. I bought 2.5m of the ribbon. How much was the cost?

0 1 2 2.5 3 (m)

Takumi
2.5m is made up of 25 0.1m pieces. So, we can find the price for 0.1m, and then find what 25 times that price is.

0 0.1 1 2 3 (m)

• Price of 0.1m = 80 ÷ 10 = 8 yen
 • Cost of 2.5m = (80 ÷ 10) × 25 = 80 × 2.5 = 80 × 10 ÷ 4 = 2000 ÷ 4 = 500 yen

Answer: yen

Reasoning with different unit Dividing by Decimals.

Miho
2.5m is made up of 25 pieces of 0.1m.

0 0.1 1 2 2.5 3 (m)

• Price for 0.1m 300 ÷ 25
 • Price for 1m (300 ÷ 25) × 10
 300 ÷ 2.5 = 300 ÷ 25 × 10 = yen

Answer: yen

Big Ideas

- ▶ Visual representations such as double number lines are (can be) powerful reasoning tools.
- ▶ Developing a reasoning tool requires time.
- ▶ Collaboration across grades is essential.



Thank you.

