Curriculum Exploration (2) Operations and Algebraic Thinking

Tad Watanabe
Kennesaw State University

Kennesaw State University
College of Science and Mathematics
Department of Mathematics

3 CCSS Clusters in 2.OA

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Even in NBT

- 2.NBT: Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Operation and Algebraic Thinking in Grade 2

- Units 3, 4, 8 & 16

Algebraic thinking in elementary schools

- In the Japanese Course of Study, what we might consider "algebraic thinking" is addressed in the domain, Quantitative Relations.
- In this domain, the main contents are "ideas of functions," "algebraic expressions and their interpretation," and "organizing and interpreting data." (*Teaching Guide*, p. 43)

Ideas of functions

- The idea of functions describes a way of thinking whereby a problem is solved by focusing on patterns of change or correspondence involving quantities or geometrical figures. It is especially important to investigate relationships between two quantities that change simultaneously, and interpret and express characteristics and tendencies of the relationship. (*Teaching Guide*, p. 43)

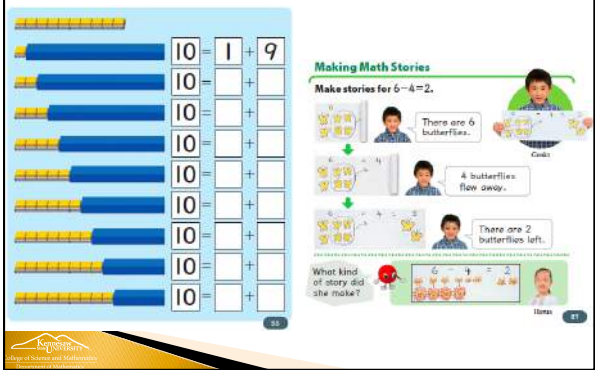
Algebraic expressions

- Algebraic expressions, or "math sentences" – includes expressions, equations and inequality.
- Algebraic expressions are sometimes called the language of mathematics, and they play important roles in expressing objects and their relationships accurately and concisely so that they can be understood. It is important to be able to interpret algebraic expressions and to use them in relationship to words and diagrams. (*Teaching Guide*, p. 43)

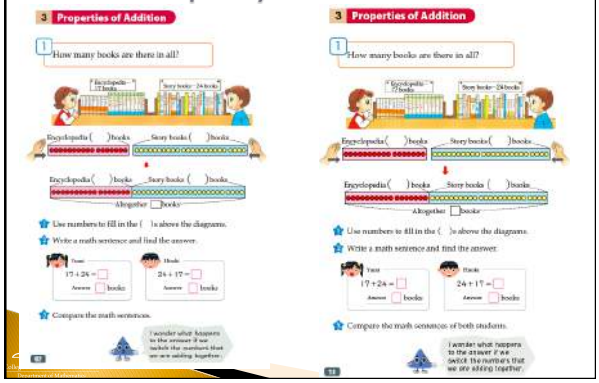
Algebraic Thinking

Grade	Ideas of functions	Algebraic expressions and their interpretation
Grade 1	• Correspondence between two objects • Sizes of numbers (large/small), order of numbers • Seeing a number as sum or difference of other numbers	• Mathematical expressions of addition and subtraction and how to interpret them
Grade 2	• Sizes of numbers (large/small), order of numbers • Seeing a number as the product of other numbers • How a product changes when the multiplier increases by 1.	• Relationship between addition and subtraction • Mathematical expressions of multiplication and how to interpret them • Mathematical expressions with ( ) and symbols like □

Kindergarten (from K textbook)



Property of Addition



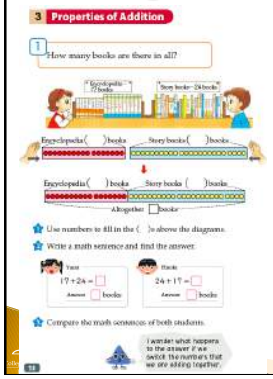
Addition & Subtraction Situations CCSS Table 1 (p. 88)

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 − 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 − ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? − 2 = 3

	Total Unknown	Addend Unknown	Both Addends Unknown[1]
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 − 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 1 + 4, 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + 3 = 5, 5 − 2 = 3	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 − 3 = ?, ? + 3 = 5

Put Together/Take Apart



- An "add together" or "put together" situation: "There are 5 blue birds and 3 white birds in a cage. How many birds are in the cage?" In this case the math sentence can be either 5 + 3 = 8 or 3 + 5 = 8. When students manipulate counting blocks to represent this part-whole addition situation, the image created is of the addition of two parts coming together (one part from the left and one part from the right side) to form the whole or total set. (TE p. 46)

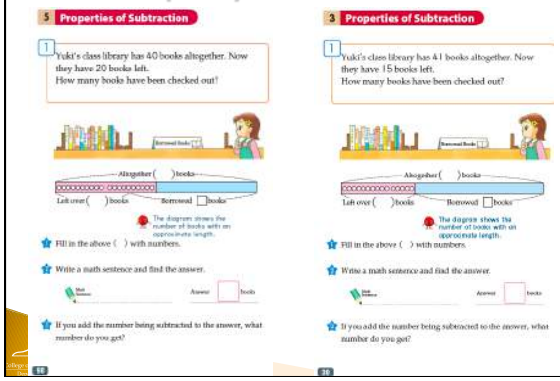
Why not "Add To" situation?

- An "add more" or "add to" situation: "There were 2 cars in a parking lot and 3 more cars drove into the lot." The math sentence that represents this situation is 2 + 3 = 5. In this case 3 + 2 = 5 does not appropriately represent the parking lot situation, because 3 + 2 = 5 means "There were 3 cars in a parking lot and 2 more cars drove into the lot."

Why not "Add To" situation?

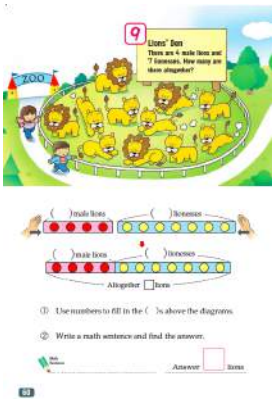
- An "add more" or "add to" situation: "There were 2 cars in a parking lot and 3 more cars drove into the lot." The math sentence that represents this situation is 2 + 3 = 5. In this case 3 + 2 = 5 does not appropriately represent the parking lot situation, because 3 + 2 = 5 means "There were 3 cars in a parking lot and 2 more cars drove into the lot."

Property of subtraction



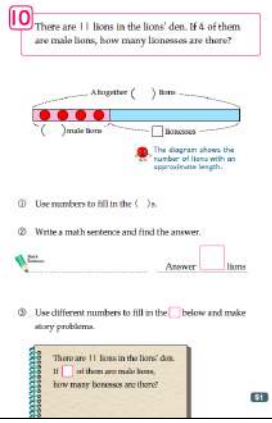
Tape Diagram

- Students first encountered a tape diagram in Grade 1 (IES Grade 1 p. 60).



Tape Diagram

- Then, a Put Together/ Take Apart situation with a part unknown problem is used to introduce a blank tape representing a (discrete) quantity.



IES Grade 1 TE

A tape diagram represents simply and clearly the relationship of quantities that appear in a story problem situation. The tape diagram also clarifies what students need to do to find the answer. It helps them decide what operations to use, what math sentence represents the situation, and why the math sentence can be used to solve the problem. Diagrams also help students explain their solution process as well as justify their solution and answers.

IES Grade 1 TE

In this lesson students represent the problem situation with counters and strips of paper tape. This lesson marks a transitional period for students in their use of diagrams to represent situations. They are transitioning from using discrete counters (or circles) to using linear tape. Therefore the diagrams used in this lesson include both forms of representations. Future lessons in the textbook will show diagrams that are gradually moving away from including semi-concrete counters and moving toward using the tape model as the only representation.

IES Grade 1 TE

It is important that students know the length of the strips of tape represent the quantity of a set and that the longer strip of tape represents a greater quantity. However, it is also important to note that in some cases it is not easy to represent the length of tape in a diagram of an unknown quantity. Therefore, the primary purpose for using a tape diagram is to show the relationship of quantities, rather than showing an accurate representation of the quantities. (p. 162)

Shifting Focus

- Representations for specific values of quantities

- Representations for relationships among quantities

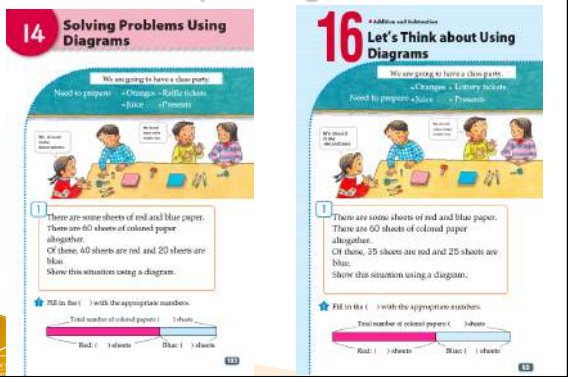
Math Int'l Grade 2 TE

- In the section where students learn to represent subtraction problem questions using tape diagrams, identifying the structure of the problem situations becomes complicated because students need to differentiate and figure out the place of the unknown quantity; for example, whether to solve for how many are left (take from, result unknown), how many were there initially (take from, start unknown), or which one has more and how many more (compare, result unknown).

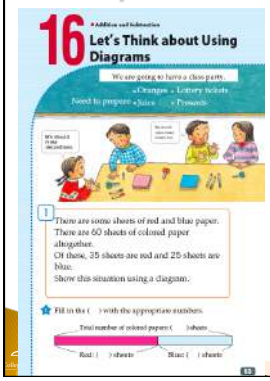
Math Int'l Grade 2 TE

- Thus, it is important for the teacher to provide appropriate support to students, such as asking students to underline the keywords in the problems and showing pictures, photographs or video to help students understand the story problem situation. Ultimately, tape diagram representations will help students see the relationship between quantities in subtraction problems. (p. 54)

Tape Diagram



Representing Relationships

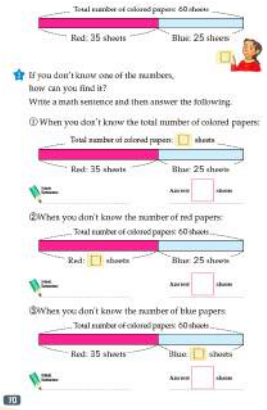


- All quantities are given because the focus is on representing how they are related.
- "altogether" and "of these" suggest this is a Put Together/Take Apart situation.

Representing Relationships

The ability to express the relationships of parts and whole quantities within problem situations and to convey this part-whole structure is critical to problem solving. The textbook attaches great importance to gradually and systematically teaching how to utilize diagrams to solve problems starting from the lower grade levels. This unit provides a foundation for solving problems by representing the relationships of problem quantities in tape diagrams and representing these relationships in math sentences. These skills are applied when students learn multiplication and division of fractions and decimal numbers in Grades 4 through 6. (TE p. 399)

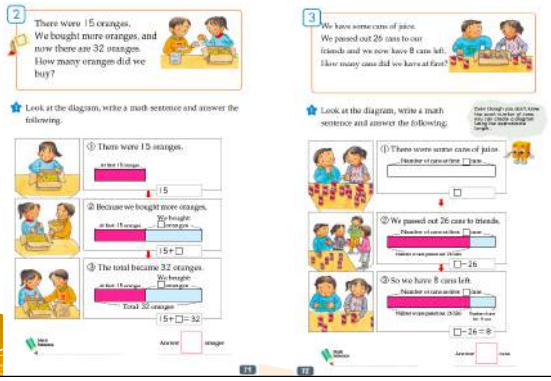
Gradual development



Addition & Subtraction

The goal of this unit is to increase students' ability to understand the inverse relationship between addition and subtraction by solving story problem situations and representing these situations in math sentences. As students interpret problem situations and math sentences, the textbook pages and the teacher support students' realization that using a tape diagram helps them to more concisely express the structure of problem situations and more easily decide which operations to use.

Addition & Subtraction



Addition & Subtraction

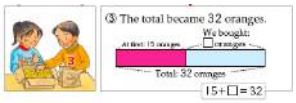
Additionally, the problems in this unit require students to reason about the inverse relationship of addition and subtraction. That is, some problem situations are presented as addition problems, but subtraction is used to solve for the unknown part. Likewise, other problem situations are presented as subtraction problems, but addition is used to solve for the unknown whole. (TE p. 399)

By substituting the unknowns with a ☐ in problems requiring reasoning about an inverse relationship, students represent the relationships among quantities through tape diagrams which correspond to the chronological order of events in problem situations. Furthermore, students decide what operations to use based on which of the three parts of their tape diagrams is the unknown. Given this sequence, students need to practice solving problem situations by substituting the unknown quantity with a ☐ in the math sentence and representing the relationship of quantities on tape diagrams before determining which operations to use to solve for the unknown in problems.

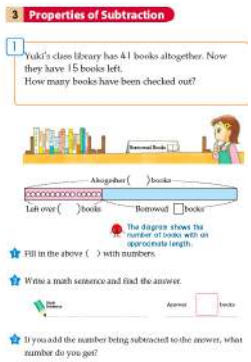
(TE p. 400)

Two types of equations

- Equations that represent situations

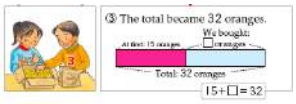


Deciding which operation



Two types of equations

- Equations that represent situations



- Equations to find the missing number

$\square = 32 - 15$

or

$32 - 15 = \square$

Explaining own reasoning

For each situation, students must explain why the respective calculations are used to find the answer. In order to explain their reasoning about the method they use, students learn to understand the relationship of quantities presented in story problems, represent the relationships and quantities in tape diagrams, and develop math sentences that represent the method used. The lessons are designed to increase students' ability to explain their mathematical reasoning using previous and newly acquired skills and to show the connection between corresponding diagrams and math sentences.

Checking Answers

2 Calculate and find the answers.
Then, calculate them again by switching the places of the numbers being added to and the numbers being added.

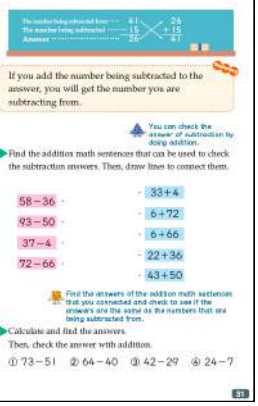
① 46+12 ② 69+25 ③ 20+36 ④ 7+59

19

The solution to an addition calculation can be checked by utilizing the inverse operation of subtraction; however, students have not learned subtraction with two-digit numbers at this point so they are encouraged instead to utilize the commutative property of addition to check their answers. (TE p. 46)

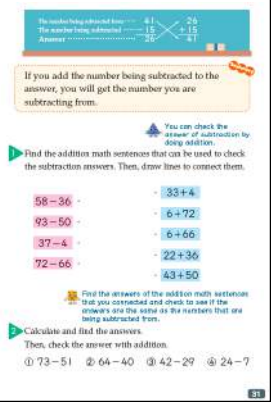
Checking Answers

In this lesson the goal is that students become aware of how the numbers in a subtraction equation are related and how they can use this relationship to check their subtraction calculations (i.e., adding the subtrahend and difference to get the minuend)…



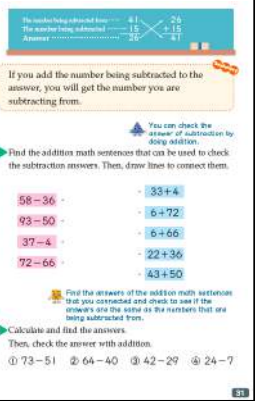
Checking Answers

Some students will have difficulty thinking about the relationship between addition and subtraction if they see and use only math sentences. An equation is a more abstract representation than counting blocks or diagrams. Help students think about the addition–subtraction relationship by developing connections among math sentences, the actual problem context and tape diagrams.



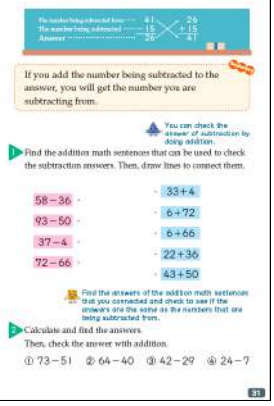
Checking Answers

When students use addition to check subtraction problems, they expand their knowledge of addition and subtraction as they demonstrate understanding and apply the bidirectional relationship of these inverse operations.



Checking Answers

Students are not expected to develop an understanding of the relationship between addition and subtraction in this lesson alone. The understanding will be further developed in Unit 16 "Addition and Subtraction," … Therefore, the learning tasks in this Lesson 8 are laying an important foundation for developing future understanding of the inverse relationship between addition and subtraction. (TE p. 72)

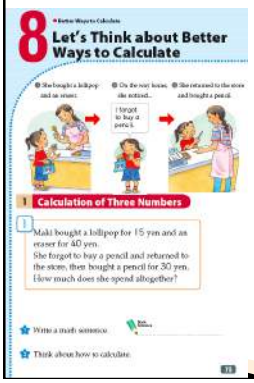


Unit 8: Better ways to calculate

- Goals

  Students deepen their understanding of addition and subtraction operations by applying properties and mental single-digit addition and subtraction calculations. Students expand their ability to utilize properties through decomposition/ composition of numbers.

Unified math sentence


8 Let's Think about Better Ways to Calculate

1 Calculation of Three Numbers

1 Maki bought a lollipop for 15 yen and an eraser for 40 yen. She forgot to buy a pencil and returned to the store, then bought a pencil for 30 yen. How much does she spend altogether?

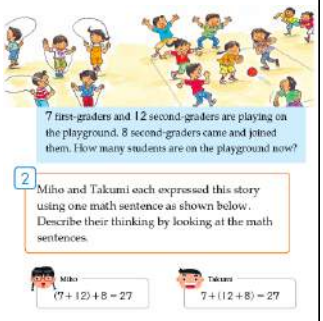
Write a math sentence.

Think about how to calculate.

- 15 + 40 = 55
  55 + 30 = 85
- Unified math sentence
  (15 + 40) + 30 = 85

Interpreting math sentences

When teaching this unit, help students recognize the merits and understand the functions of math sentences (expressions and equations) through activities where they are asked to represent situations and interpret math sentences. (TE p. 171)


7 first-graders and 12 second-graders are playing on the playground. 8 second-graders came and joined them. How many students are on the playground now?

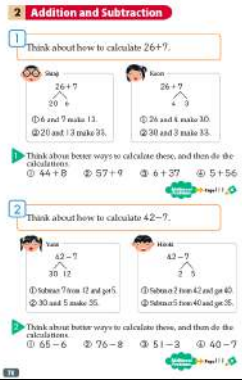
2 Miho and Takumi each expressed this story using one math sentence as shown below. Describe their thinking by looking at the math sentences.

Miho (7 + 12) + 8 = 27

Takumi 7 + (12 + 8) = 27

Thinking about ways to calculate

In this unit, it is a good idea for the teachers to nurture students' habit of studying numbers before calculating and encourage them to continue this deliberate practice in the future. (TE p. 171)


2 Addition and Subtraction

1 Think about how to calculate 26+7.

Think about better ways to calculate these, and then do the calculations.
① 44+8 ② 57+9 ③ 6+37 ④ 5+56

2 Think about how to calculate 42−7.

Think about better ways to calculate these, and then do the calculations.
① 65−6 ② 76−8 ③ 51−3 ④ 40−7

Practice Problems

Additional Problems → Page 111 X

8 Let's Think about Better Ways to Calculate Page 75~78 Answers → page 112

Page 76 1 → W Look carefully at the numbers, think about how to calculate and do the calculations.
① 3+14+6 ② 28+31+9 ③ 18+7+32

Page 78 1 → X Think about better ways to calculate, and then do the calculations.
① 89+9 ② 38+7 ③ 4+39 ④ 5+77

Page 78 2 → Y Think about better ways to calculate, and then do the calculations.
① 92−4 ② 33−7 ③ 41−5 ④ 90−8

Treasure Hunting in TE

Support **Reading story problem situations**

When the text of story problem situations is lengthy (as in Problems 2 and 3), some students will find it difficult to comprehend the content of the problems. It is helpful to suggest that students retell the story in their own words, underline important words and/or circle numbers in the story problems.

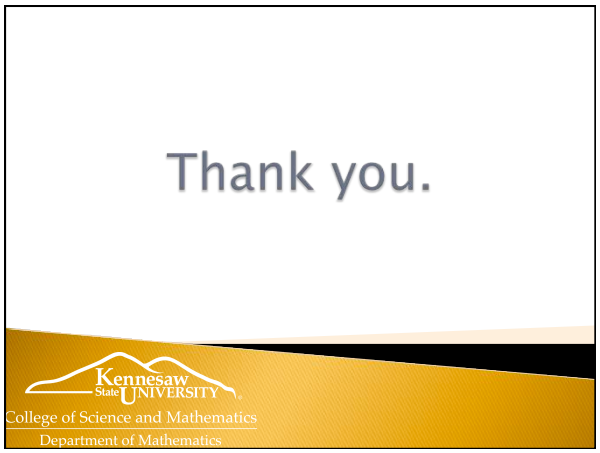
TE p. 48

How to do mental calculations

Mental calculations usually start from the highest place value. However, in this lesson, students will learn a more general way of doing mental calculations. Students are instructed to think about better ways to calculate mentally by decomposing the augend or addend and the minuend or subtrahend so their calculations become flexible and efficient. First, teachers should recognize students' many different calculation ideas and ask them to do the process of calculation mentally. Then as students practice calculations, help them look more carefully at the numbers involved in the calculations and think about what might be a more effective and efficient method to use for those numbers. In this way students will develop a method that is the most effective and efficient way for them to solve mental calculation problems.

TE p.180

As children practice solving problem situations, they eventually become accustomed to solving problems using inverse reasoning and tape diagrams, enabling them to quickly decide which operation(s) to use for solving problems. However, when problem solving becomes routine, the process becomes mechanical and often students forget the meaning of the math sentences. When solving a problem situation, be sure to do the following: ask why students chose to use a particular operation/calculation; incorporate mathematical activities that require an explanation using the relationship between math sentences and tape diagrams; and confirm that they not only understand the procedure, but also the meaning and/or inverse relationship or properties that form the basis of their calculations. (TE p. 400)


Thank you.
Kennesaw State University
College of Science and Mathematics
Department of Mathematics