Curriculum Exploration (1) Number and Operations in Base 10

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2 Clusters in 2.NBT

- Understand place value (Units 6 & 14)
- Use place value understanding and properties of operations to add and subtract. (Units 3, 4 & 9)

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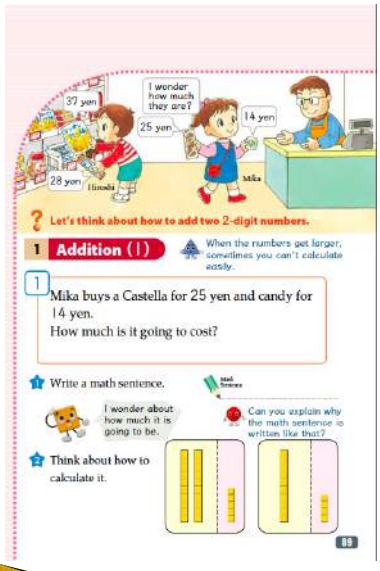
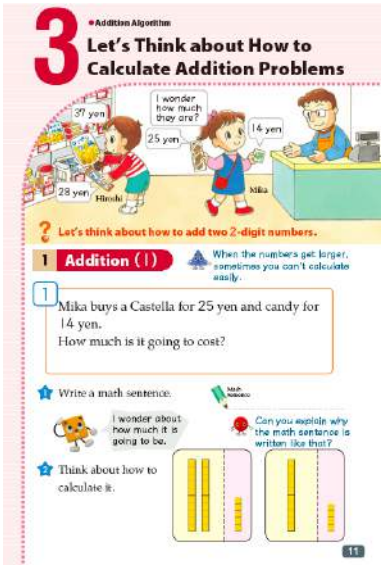
IES Grade 1



Math Int'l Grade 2



IES Grade 1

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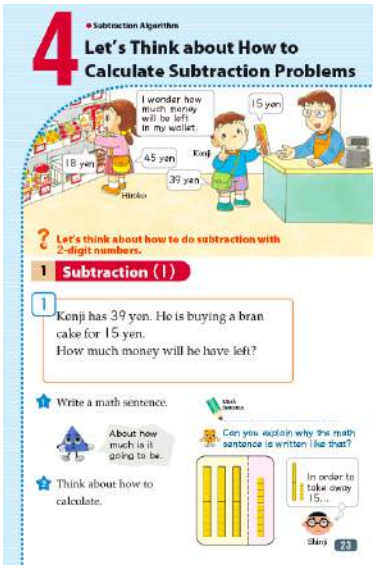

Unit Goals

- Unit 3: Students understand how to add two-digit numbers accurately using an algorithm. Students deepen their understanding of addition by enhancing their use and understanding of the algorithm. (TE p. 23)

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Unit Goals

- Unit 4: students understand how to subtract two-digit numbers reliably using the standard algorithm that draws from students' knowledge of place value and tens. As they apply the algorithm, they will explain their reasoning and deepen their understanding of subtraction. (TE, p. 54)

Formative Assessment

- Interest and Disposition

  Students recognize the merit of calculating using an algorithm for addition of two-digit numbers. They actively and fluently utilize a consistent calculation method in school and in their daily lives.

- Mathematical Reasoning

  Students think about and express the process of using an addition algorithm, by representing the process in diagrams and math sentences.

Formative Assessment

- Skills and Procedures

  Students calculate the addition of two-digit numbers using an algorithm and demonstrate a conceptual understanding of the calculation process.

- Knowledge and Understanding

  Students know that the addition algorithm for calculating two-digit numbers utilizes the basic addition calculation process as applied to calculating one digit numbers. Their work demonstrates an understanding of the process of calculating addition within the algorithm.

About Unit Goals

- When teaching this unit, it is important to understand that the goals go beyond helping students understand an algorithm procedure for calculating addition. (TE p. 25 – Unit 3)
- In this unit students realize that they can subtract with two-digit numbers by applying what they learned previously about the calculation of one-digit numbers and the structure of base ten numbers. (TE p.53 – Unit 4)

Ideas to be emphasized

3 **Ideas to be emphasized:**

- Thinking based on previous knowledge: In this unit, students will think about how to calculate subtraction using the place value algorithm. Students pay attention to place value to reason about the algorithm calculation process. They use the number blocks to think about and understand the calculations by connecting how number blocks are moved and how the algorithm proceeds. Through this process, the students deepen their understanding of the subtraction algorithm calculation.
- Estimating the results of calculations: Students are also asked to estimate answers before calculating subtraction problems. They learn to recognize obvious mistakes in calculations and enhance their ability to think logically.
- Systematically expanding the method of the standard algorithm: The standard algorithm is an efficient and effective calculation method for subtraction. Students start with basic subtraction problems that have the same number of digits in the minuend as the subtrahend (e.g., 39 - 15) to develop an understanding of how to calculate using the algorithm. Students then learn to calculate more difficult subtraction problems, such as 37 - 20 (the subtrahend is a multiple of 10 with zero in the ones place) and 37 - 5 (the subtrahend is a one-digit number). When students think about the similarities and differences among these different types of subtraction problems, they are using their analogical thinking and expanding their understanding of the standard subtraction algorithm.
- Checking answers: In the final lessons of this unit students study the relationship between addition and subtraction. By thinking about the numbers in a problem situation related to borrowing books in a library, students learn that when the subtrahend is added to the difference (answer) in a subtraction problem, the sum becomes the minuend. Thereafter, students use the inverse relationship between addition and subtraction to check answers to subtraction calculations.

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Ideas to be emphasized

- Thinking based on knowledge previously learned.
- Estimating the results of calculation.
- Analyzing different types of calculation problems.
- Checking answers.

Applying previous learning

- A major emphasis in the Japanese curriculum.
- Look for and make use of structures.
- Look for and express regularity in repeated reasoning.

What students have learned:

- Single-digit addition that require re-grouping, e.g., 7 + 4.
- Addition of multiples of ten, e.g., 50 + 20.
- Addition of a two-digit number with a single-digit, e.g., 25 + 3.

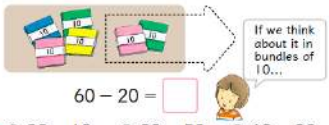
Addition and Subtraction

1 How many pieces of colored paper are there altogether?

$50 + 20 =$ ☐

2 ① $10 + 40$ ② $50 + 30$ ③ $20 + 70$
④ $30 + 70$ ⑤ $60 + 40$ ⑥ $10 + 90$

3 There are 60 pieces of colored paper.
If we use 20 pieces, how many will be left?

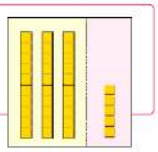


$60 - 20 =$ ☐

4 ① $30 - 10$ ② $90 - 50$ ③ $60 - 30$
④ $100 - 20$ ⑤ $100 - 60$ ⑥ $100 - 30$

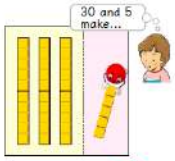
48

5 35 is 30 and 5.
Write the number in the ☐.



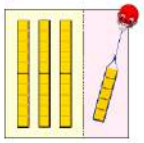
① the number when 5 is added to 30

$30 + 5 =$ ☐



② the number when 5 is subtracted from 35

$35 - 5 =$ ☐



6 ① $40 + 2$ ② $60 + 8$ ③ $50 + 3$
④ $42 - 2$ ⑤ $73 - 3$ ⑥ $84 - 4$

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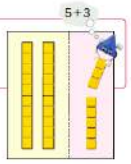
Grade 1

7 Think about how to calculate 25+3.



$25 + 3 =$ ☐

8

① $45 + 3$	④ $23 + 4$
② $65 + 3$	⑤ $53 + 4$
③ $75 + 3$	⑥ $63 + 4$

9 Think about how to calculate 28−3.

$28 - 3 =$ ☐

10

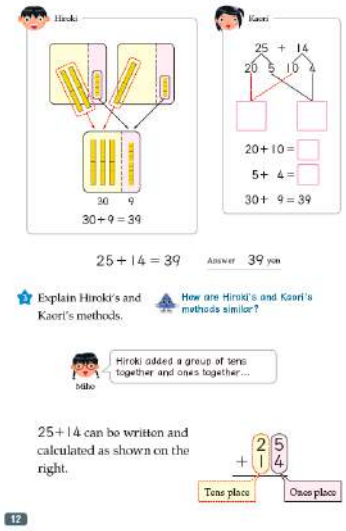
① $48 - 3$	④ $27 - 4$
② $68 - 3$	⑤ $57 - 4$
③ $78 - 3$	⑥ $67 - 4$

48

Grade 1

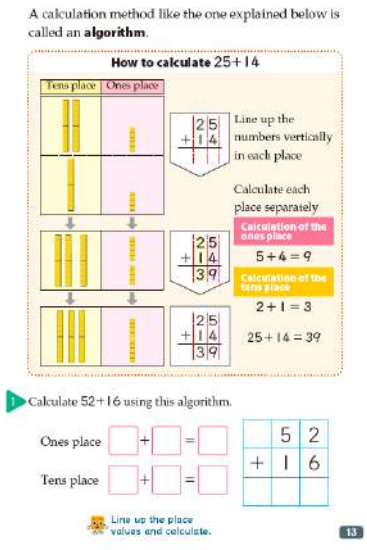
Grade 2

- Building on students' understanding that we add tens with tens and ones with ones – introduce the vertical notation.



Thinking about and explaining

- Instruction in this unit is focused on helping students explain the process of finding answers (sums), including how an algorithm calculation is processed. (TE p. 26)



Supporting Students

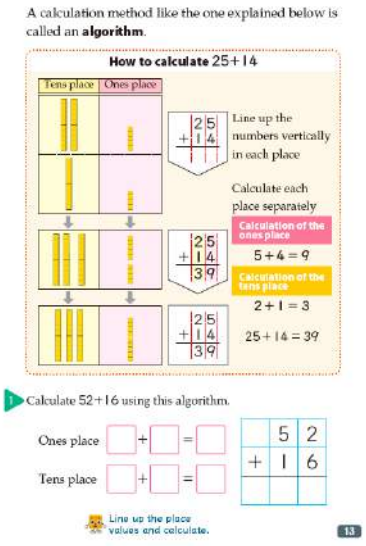
Support

Accommodations for students who are struggling:

Some students have difficulty memorizing the process of calculation and easily forget the process or steps of an algorithm, especially if they don't understand the mathematics of the algorithm. In this unit, students learn the process of two-digit plus two-digit addition by decomposing numbers into tens and ones before conducting calculations. Therefore, it is very important for students to use the counting blocks to practice calculating so they can develop a visual image of the calculations. This is a necessary step to understanding the calculation steps of the addition algorithm. To aid students' understanding of the calculation process, it is recommended that students work with counting blocks and are provided with a place value chart that is color coded (i.e., pink for the ones place and yellow for the tens place, as depicted in the student textbook). These tools offer a helpful and very simple visual calculation process guide. A final suggestion for struggling students is to provide them with grid paper to practice the algorithm calculations.

Working with Blocks

- Therefore, it is very important for students to use the counting blocks to practice calculating so they can develop a visual image of the calculations. This is a necessary step to understanding the calculation steps of the addition algorithm.


A calculation method like the one explained below is called an **algorithm**.

How to calculate 25+14

Tens place | Ones place

25 + 14

Line up the numbers vertically in each place

Calculate each place separately

Calculation of the ones place: 5+4=9

Calculation of the tens place: 2+1=3

25+14=39

Calculate 52+16 using this algorithm.

Ones place ☐ + ☐ = ☐

Tens place ☐ + ☐ = ☐

	5	2
+	1	6

Line up the place values and calculate.

13

Working with Blocks

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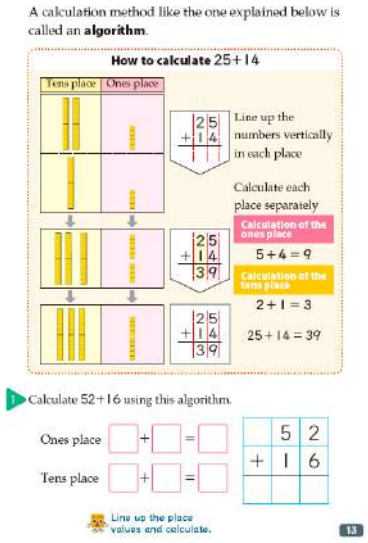
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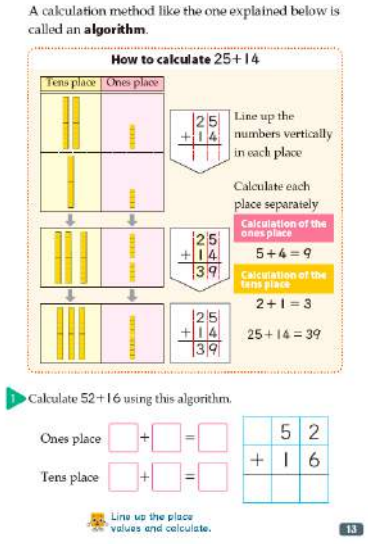
Working with Blocks

- A final suggestion for struggling students is to provide them with grid paper to practice the algorithm calculations.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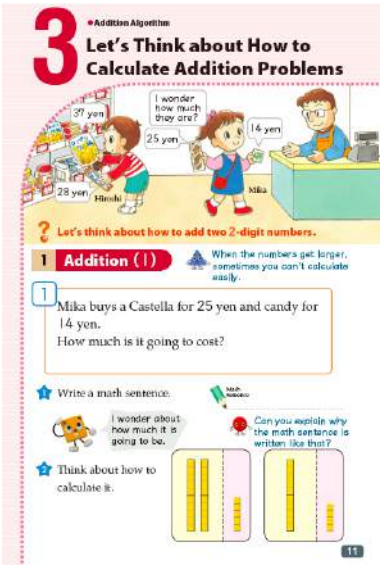
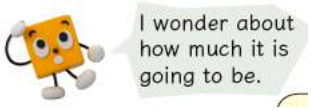
Estimating

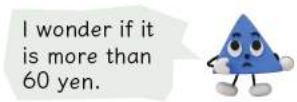
- Students estimate the result of a calculation before actually calculating the sum. This practice helps students not only recognize obvious mistakes in calculations, but also enhances their ability to think logically.



Estimating 25 + 14

- How can children use what they have learned to estimate the result of 25 + 14?





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Different types of calculation

- The "basic type"

	5	2
+	1	6

- Special cases

① 30 + 24 ② 30 + 20

① 32 + 4 ② 6 + 50

Merit of calculating from the ones place (TE p. 34)

About the merit of calculations starting from the ones place

Even if the calculations are performed starting from the larger "tens" place, it is unlikely that these addition calculations without regrouping will be difficult for students.

When thinking about how to calculate 25 + 14 using counting blocks, many students start by moving blocks from the tens place for addition calculations without regrouping. This indicates that students do not see an advantage to starting the calculation from the ones place. However, when using the algorithm calculation with regrouping, the calculation process becomes more difficult if the calculation starts from the larger place value.

$\begin{array}{r} 52 \\ +\ 16 \\ \hline 68 \end{array}$ ← Even if we start calculating from the tens place, the calculation is still easy.

$\begin{array}{r} {\scriptstyle 1} \\ 37 \\ +\ 28 \\ \hline 55 \\ 6 \end{array}$ ← If we start calculating from the tens place 3 + 2 first, we get 5. We then need to cross out the 5 and change it to 6 because a regrouping occurs in the ones place.

The merit of starting the calculations from the ones place will be discussed in Lessons 4 and 5, when students learn algorithm calculations with regrouping.

- The merit of starting the calculations from the ones place will be discussed in Lessons 4 and 5, when students learn algorithm calculations with regrouping.

Common mistakes (TE p. 36)

- **Examples of frequent student mistakes**

  < Places are not aligned vertically >

  $\begin{array}{r} 30 \\ +\ 24 \\ \hline 270 \end{array}$ $\begin{array}{r} 32 \\ +\ 4 \\ \hline 72 \end{array}$ $\begin{array}{r} 6 \\ +\ 50 \\ \hline 110 \end{array}$

  < No zero in ones place >

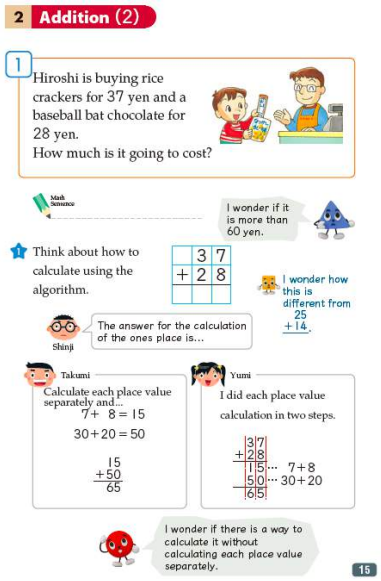
  $\begin{array}{r} 30 \\ +\ 20 \\ \hline 5 \end{array}$

- **How to reduce students' calculation mistakes**

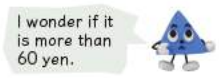
  When in the process of algorithm calculations students who write their numbers sloppily or who ignore grid lines in their notebooks often make calculation mistakes. Therefore establish the following rules for calculating:

  • Write letters and numbers neatly. Write only one number in each square of the grid paper

Addition with re-grouping



Estimating



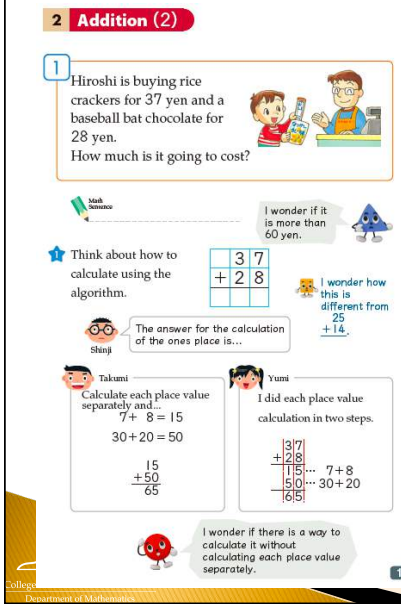
[Anticipated responses and support]

a. About 60 yen, because we have two items that are almost 30 yen each.

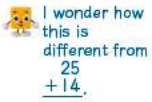
b. Because 37 yen is about 40 yen, and 28 yen is about 30 yen, the total is about 70 yen.

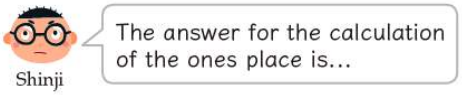
➡ Be sure to ask students to clarify how they look at and think about the numbers to estimate the answer.

Addition with re-grouping



Connecting to previous learning





[Anticipated responses and support]

a. When you add the numbers in the ones place, the answer becomes greater than 10.

b. The calculation has a regrouping.

➡ Help students pay attention to the addition calculation of the two numbers in the ones place and the answer.

Think about ways to calculate

2 Independent problem solving

● ★1 Students think about how to do the algorithm calculation for 37 + 28.

Hatsumon	Let's think about how to do the algorithm calculation for 37 + 28!

• Ask students who are having difficulty calculating to use counting blocks to think about the calculation.

Focus on the process, not the answer

3 Discussion

● Students present their ideas and understand each other's ideas.

Hatsumon	How did you calculate and find the answer for 37 + 28 as 65?

[Anticipated responses and support]

a. Calculate each place value separately and... (Takumi's idea)
b. I did each place value calculation in two steps.
➡ Help students understand that both ideas use the same idea of calculating each place separately.
➡ If idea b was not presented by a student, presents the idea and ask students to think about how the calculation was done with this method.

Yumi's idea

Yumi

I did each place value calculation in two steps.

$$\begin{array}{r} 37 \\ +\ 28 \\ \hline 15 \cdots\ 7+8 \\ 50 \cdots\ 30+20 \\ \hline 65 \end{array}$$

■ **The purpose of showing Yumi's idea**

The reasons why Yumi's idea was pictured in the textbook include the following: (1) It is easier to understand the calculation process when each place is calculated separately; and (2) Yumi's idea is a very useful method for students to think about regrouping calculations and where to place the newly composed ten.

If Yumi's idea is not presented by a student, present this idea and ask students to think about how the calculation was regrouped to the tens using this method.

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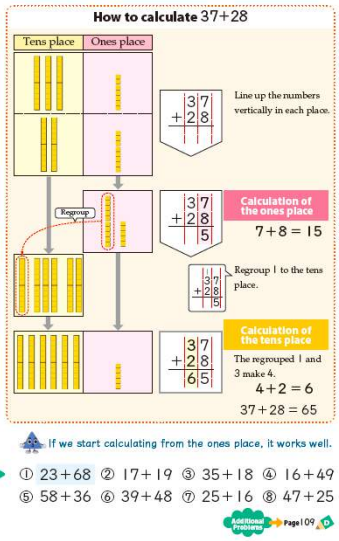
Merit of calculating from the ones place



Hatsumon Why do you think it is a good idea to start the calculation from the ones place first?

- As we summarize the lesson, have students think about the merit of starting the calculation in the ones place.

Practice is important, but…

- TE p. 50
- If students are assigned many practice problems all at once, they may develop an incorrect procedure for calculating with regrouping because they may not completely understand the place value algorithm calculation process. This practice will reinforce the wrong or inaccurate procedure. Therefore, it is very important to assign only one problem at first and carefully observe how each student understands the algorithm calculation process. This way the teacher can make a more informed decision to support each individual student by assigning the appropriate amount and level of practice that meets each student's individual needs.

Different types of calculation

- "0" in the sum

$$\begin{array}{r} 28 \\ +\,32 \\ \hline \end{array}$$

- Missing place

$$\begin{array}{r} \textcircled{1}\,28 \\ +\,\square 6 \\ \hline \end{array} \qquad \begin{array}{r} \textcircled{2}\,28 \\ +\,\square 2 \\ \hline \end{array}$$

3-digit sum (Unit 9)

- re-grouping in the tens place

$$\begin{array}{r} 83 \\ +\,46 \\ \hline \end{array}$$

- re-grouping in the tens and ones places

$$\begin{array}{r} 76 \\ +\,58 \\ \hline \end{array}$$

- re-grouping in the tens and ones places (2)

$$\begin{array}{r} \textcircled{1}\,46 \\ +\,57 \\ \hline \end{array} \qquad \begin{array}{r} \textcircled{2}\,93 \\ +\,\square 8 \\ \hline \end{array}$$

Different types of calculation

- By thinking about the similarities and differences of various types of calculation problems, students use analytical thinking to reason about the calculation process and expand their understanding of standard addition algorithm calculations. (TE p. 26)

Unit 4: Subtraction

- The flow of the unit is very similar to Unit 3.
- There are many similar features in TE.

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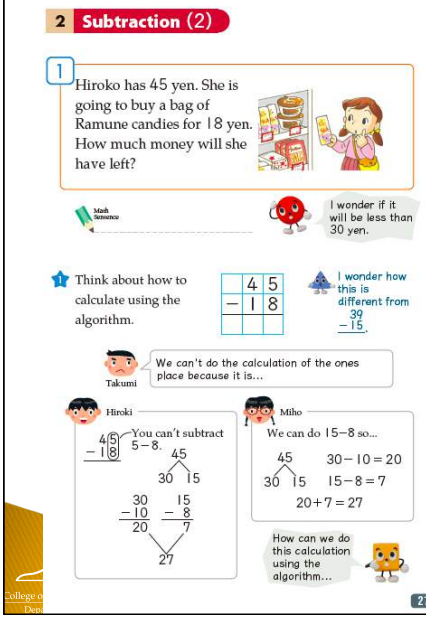
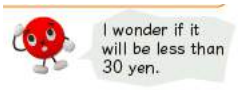
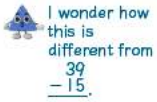
Subtraction with re-grouping



Estimating



Connecting to previous learning



Starting with the basic type

	4	5
−	1	8

Two ways of reasoning

- Grade 1 (e.g., 12 – 7)
  - Minuend decomposition method (subtract-add)

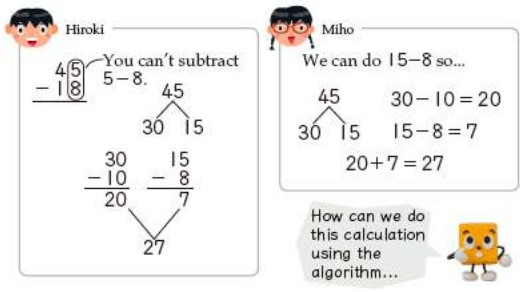
    12 = 10 + 2

    12 – 7 = 10 – 7 + 2
  - Subtrahend decomposition method (subtract-subtract)

    7 = 2 + 5

    12 – 7 = 12 – 2 – 5

Emphasis on minuend decomposition method



- Which method are they using?

About these two methods
(TE p. 64)

It is important to recognize different ways subtraction problems can be calculated. In this particular lesson, the textbook page and discussion focuses on regrouping by decomposing the minuend. However, students may use the subtrahend decomposition method to solve the problems. Recognize these students' methods to widen all students' mathematical view and thinking.

Compared to the minuend decomposition method, however, the subtrahend decomposition method is not easy to compute nor is it simple to understand. It is important that students understand this distinction as the class compares methods and sees how the minuend decomposition method is the easier and simpler calculation method.

Starting from the ones place

$$\begin{array}{r} 39 \\ -\ 15 \\ \hline 24 \end{array}$$

Even if we start calculating from the tens place first, the calculation is still easy.

$$\begin{array}{r} 45 \\ -\ 18 \\ \hline 37 \\ 2 \end{array}$$

If we start calculating from the tens place 4 - 1 first, and get 3, because we can't subtract 8 from 5, we need to regroup. So we need to carry down 10 from the 3 in the tens place. The calculation becomes very complicated.

The merit of starting the calculation from the ones place will be discussed in Lessons 4 and 5 when students learn algorithm calculations with regrouping.

TE p. 60

Additional practice problems

Subtraction problems for those who accurately and quickly finish the problems in the lesson:

❶ Calculate the following. (Ⓑ type)

(1)	(2)	(3)	(4)	(5)
80 − 65 = 15	70 − 13 = 57	90 − 37 = 53	50 − 36 = 14	50 − 18 = 32

❷ Calculate the following. (Ⓒ type)

(1)	(2)	(3)	(4)	(5)
81 − 74 = 7	58 − 49 = 9	33 − 25 = 8	27 − 18 = 9	92 − 85 = 7

❸ Calculate the following. (Ⓓ type)

(1)	(2)	(3)	(4)	(5)
26 − 8 = 18	81 − 2 = 79	46 − 9 = 37	62 − 7 = 55	37 − 9 = 28

❹ Calculate the following. (Ⓔ type)

(1)	(2)	(3)	(4)	(5)
70 − 8 = 62	50 − 3 = 47	90 − 7 = 83	80 − 5 = 75	20 − 2 = 18

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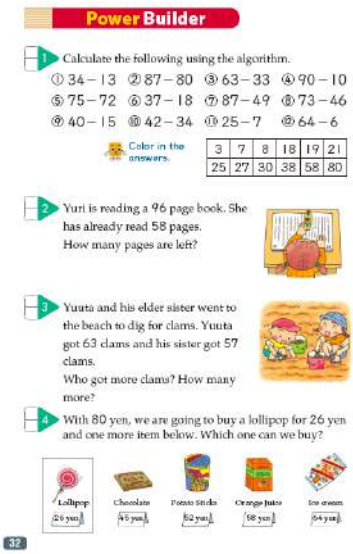
Unit outline

Sub-Units	Lesson	Pages	Primary Learning Content
1. Subtraction (1)	1	23 - 24	• Think about how to subtract two-digit numbers.
	2	25 -26	• Use an algorithm to calculate two-digit minus two-digit = two-digit problems without regrouping.
	3	26	• Use the algorithm calculation for two-digit minus one-digit = two-digit problems without regrouping, or two-digit minus two-digit = two-digit or one-digit problems without regrouping.
2. Subtraction (2)	4	27	• Think about how to calculate subtraction with two-digit numbers with regrouping.
	5	28	• Use the algortihm to calculate two-digit minus two-digit = two-digit problems with regrouping.
	6	29	• Use the algorithm to calculate two-digit - one-digit = two-digit problems with regrouping, or two-digit minus two-digit = two-digit or one-digit problems with regrouping.
	7	29	• Create problems with two-digit minus one-digit = two-digit problems with regrouping or two-digit minus two-digit = two-digit problems with regrouping.
3. Properties of Subtraction	8	30 - 31	• Use the inverse relationship between addition and subtraction to solve problems and check solutions.
Summary	9	32	• Deepen understanding of math content (Power Builder).
	10	33	• Deepen understanding of math content (Mastery Problems).

Summary: Power Building



Power Builder

1 Calculate the following using the algorithm.

① 34 − 13 ② 87 − 80 ③ 63 − 33 ④ 90 − 10
⑤ 75 − 72 ⑥ 37 − 18 ⑦ 87 − 49 ⑧ 73 − 46
⑨ 40 − 15 ⑩ 42 − 34 ⑪ 25 − 7 ⑫ 64 − 6

Color in the answers.

3	7	8	18	19	21
25	27	30	38	58	80

2 Yuri is reading a 96 page book. She has already read 58 pages. How many pages are left?

3 Yuuta and his elder sister went to the beach to dig for clams. Yuuta got 63 clams and his sister got 57 clams. Who got more clams? How many more?

4 With 80 yen, we are going to buy a lollipop for 26 yen and one more item below. Which one can we buy?

Lollipop 26 yen | Chocolate 45 yen | Potato Sticks 52 yen | Orange Juice 58 yen | Ice cream 64 yen

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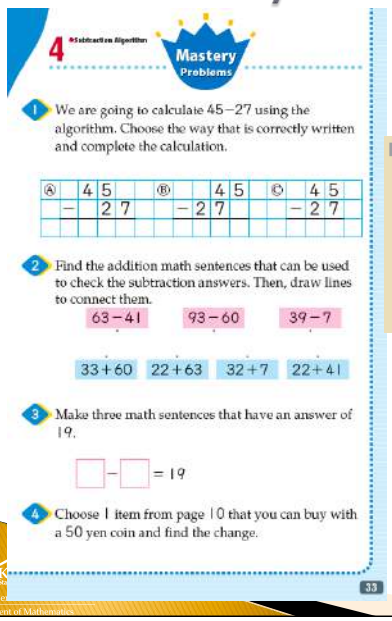
About "Power Builder" problems

The goal of "Power Builder" problems is to help students acquire and retain the basic knowledge and skills they have learned in this unit. Students indicate they have both solved and checked their solutions to Problems 1, 2, 3 and 4 by putting a check mark in the boxes adjacent and to the left of the problem numbers (inside the green triangles). When students solve the problems a second time, they are given the opportunity to improve their understanding and calculation skills.

Students put check marks in the boxes when they solve the problems correctly the first time they attempt to solve them. The second check mark indicates they checked to make sure their answers were correct. On another day students are given time to solve the unchecked problems. If they solve these problems correctly, they indicate this by putting check marks in the boxes. In this way students identify and become aware of the types of problems that are more difficult for them. They actively monitor their own progress toward understanding these problem types. The system of checks is an efficient and effective way to help students review and assess their understanding.

Summary: Mastery Problems



4 Subtraction Algorithm — Mastery Problems

1. We are going to calculate 45−27 using the algorithm. Choose the way that is correctly written and complete the calculation.

Ⓐ	Ⓑ	Ⓒ
45	45	45
− 27	− 27	− 27

2. Find the addition math sentences that can be used to check the subtraction answers. Then, draw lines to connect them.

63−41 93−60 39−7

33+60 22+63 32+7 22+41

3. Make three math sentences that have an answer of 19.

□ − □ = 19

4. Choose 1 item from page 10 that you can buy with a 50 yen coin and find the change.

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- **About "Mastery Problems"**

"Mastery Problems" contain problems that capture the basics of the unit. These problems require students' mathematical reasoning and expressive ability. Use this set of problems to assess students' understanding.

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Sample blackboard organization

Date

Hiroko has 45 yen. She is going to buy a bag of Ramune candies for 18 yen. How much money will she have left?

Math Sentence: 45 - 18

- 45 - 20 = 25 About 25 yen.
- 50 - 20 = 30 About 30 yen.

☆ What is different from the previous calculation?

Saku • The calculation in the ones place is 5 - 8 so we can't subtract.

Mako • If we split 45 into....

Let's think about how to calculate 45 - 18 using the algorithm.

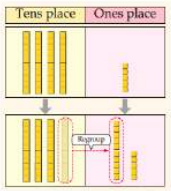
Miho We know how to calculate 15 - 8.

45
30 15
30 - 10 = 20
15 - 8 = 7
30 + 7 = 27

45
- 18
We don't know how to write regrouping.

Kai Subtract and subtract

45 - 5 = 40
40 - 10 = 30 — Too many steps!
Split 30 into 20 and 10.
10 - 3 = 7
20 + 7 = 27



Regroup one ten

3
~~4~~5
+ 18
27

Summary
- When you cannot subtract, regroup 1 from the larger place.
- Start calculating from the ones place.

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Treasure hunting in TE

- **How to facilitate student discussion**

  Provide frequent opportunities for students to participate in classroom discussions. It is recommended that students are responsible for explaining their thinking and solution process as they manipulate counting blocks. However, it is not effective when one student explains every step of his/her thinking process. Instead, ask students to explain other students' calculations or ask students to explain the calculation processes represented in math sentences or diagrams.

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To be continued…

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Base-10 Number System

- Positional system → where a digit is changes what number the digit represents
- Consistent grouping by 10 → place values that are powers of 10

Base-10 Number System

- Positional system → where a digit is changes what number the digit represents
- Consistent grouping by 10 → place values that are powers of 10
- One and only one digit in each place (except for the leading zeros of whole numbers and trailing zeros of decimal numbers) → need to re-group

If we can have more than one digit in one place,

- 453 + 172 = 5125 (five hundred twelve-ten five)
- 37 can be interpreted as having both 3 and 7 in the ones place.

Addition and Subtraction with re-grouping

- An important step in helping students understand that there must be "one and only one digit in each place."
- This rule and the use of '0' as a place holder makes it easier to read numbers written in the base-10 number system.