

# Curriculum Exploration (5b)

## Area Formulas (Grades 3 – 6)

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Department of Mathematics

## CCSS and Math International

Grade	CCSS	Math International
3	Understand concepts of area and relate area (of rectangles) to multiplication and to addition.	
4	Apply the area and perimeter formulas for rectangles in real world and mathematical problems.	Unit 11: How to measure and express area (of rectangles)
5		Unit 11: Area of quadrilaterals and triangles
6	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes;	Unit 1: Area of circles

## General Sequence of Measurement Instruction

- ▶ Direct comparison
- ▶ Indirect comparison
- ▶ Measuring with non-standard units
- ▶ Measuring with standard units

## 4 Steps of Measurement

- ▶ Understand the attribute
- ▶ Select an appropriate unit
- ▶ Cover or iterate with the unit
- ▶ Report the measurement

## Additive & Non-Additive Attributes

Additive Attributes	Non-Additive Attributes
Length Liquid Volume (Capacity) Elapsed Time Weight Angle Area Volume	Temperature Speed Density etc.

## Measurement Tools

- ▶ Learning measurement is not the same thing as learning how to use measurement tools such as rulers and protractors.
- ▶ Student-constructed measurement tools – perhaps with non-standard units – may be helpful for students to know why/how the standard tools work.

## Measurement Tools for Area



Planimeter (Platometer)

## Measuring Tools for Area

- ▶ Ways to calculate the area using the dimensions of the given figure.
- ▶ Should be the focus of instruction AFTER students understand area as a measurable attribute.
- ▶ Student should also understand the merit and necessity of quantifying the size of space (area) – that is, expressing area using numbers.

# Foundational Activity

## Play Rock, Paper, Scissors



● If you win, color in a .



Color in 1 square.



Color in 2 squares.



Color in 3 squares.



● Whoever colors more spaces wins.



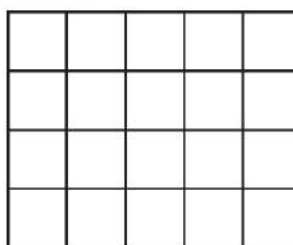
I wonder who won.



Kazuya



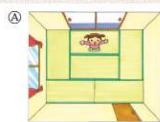
Nao



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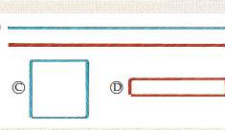
# Grade 4 Unit 11

Which has more space?



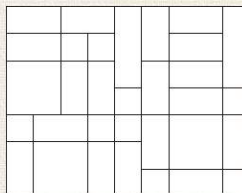
The shapes of the rooms are different, but ...

The lengths of the ropes are the same, but ...



Let's play a game using the picture below.



● Play Rock-Paper-Scissors. The person who wins will shade in a rectangle and it becomes his or her territory. The first time you win, you can pick any rectangle. After that, you can only capture a rectangle that is adjacent to your territory.  
● The person whose territory is the largest wins the game.



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## Which has more space?

**Which has more space?**

**A**  **B** 

The shapes of the rooms are different, but ...

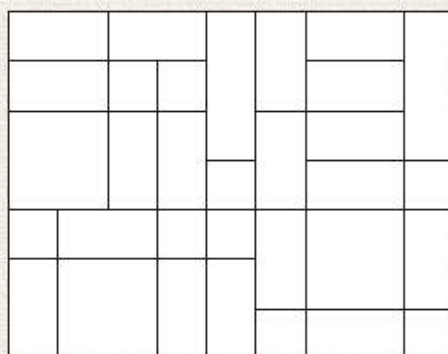
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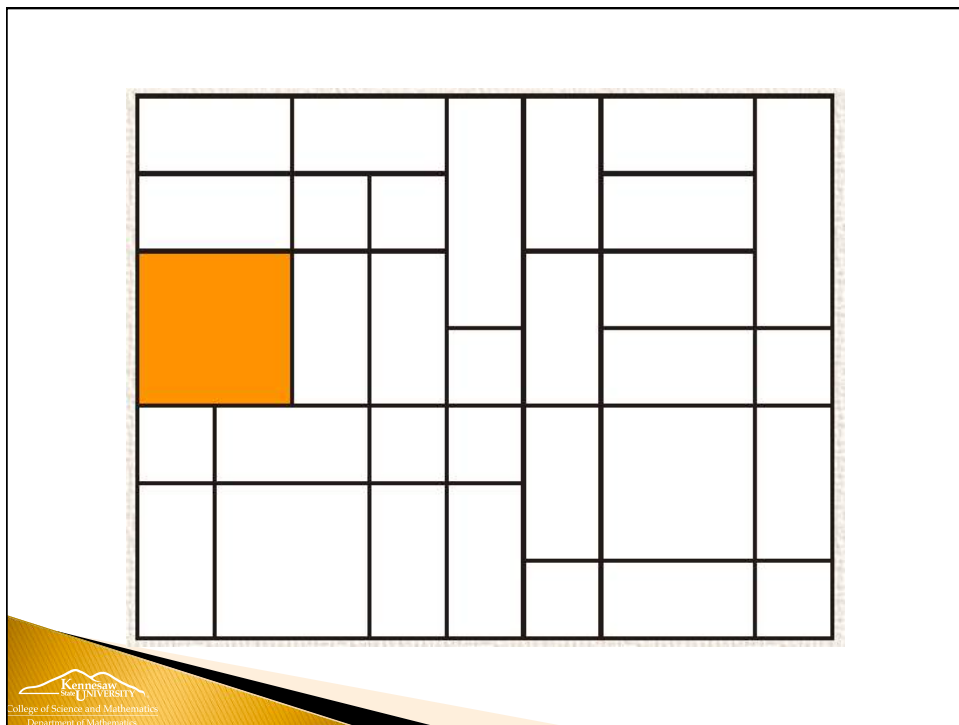
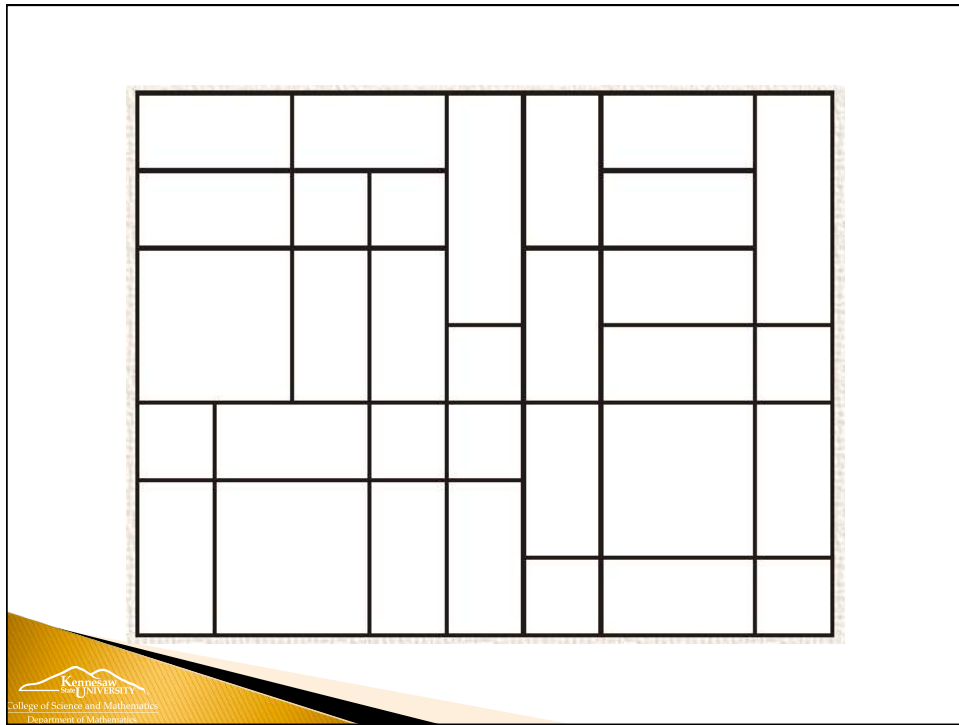
**C**  **D** 

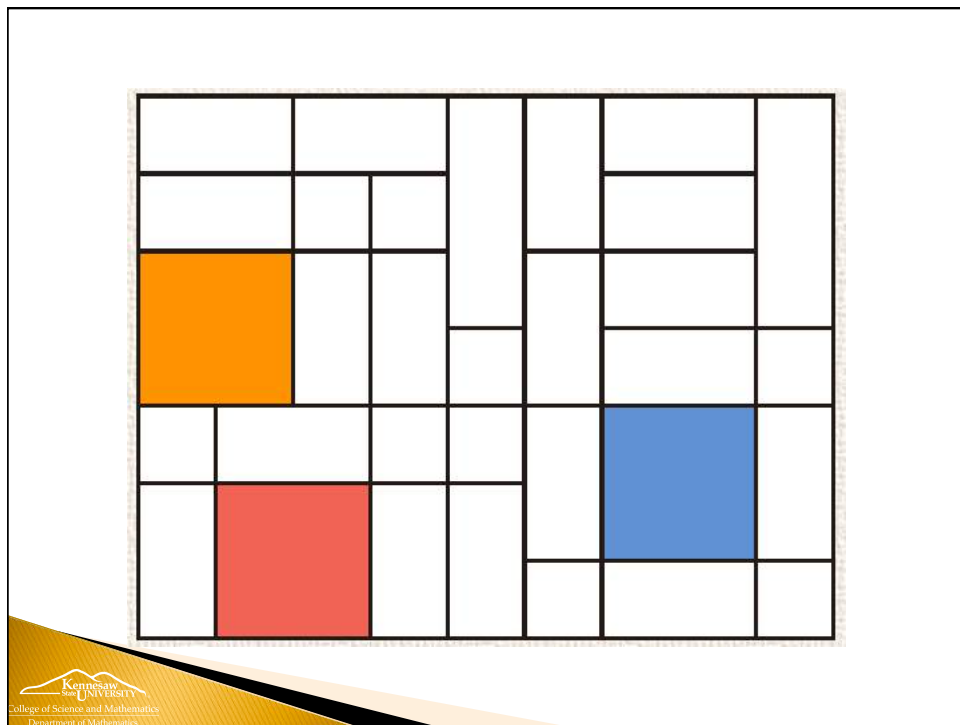
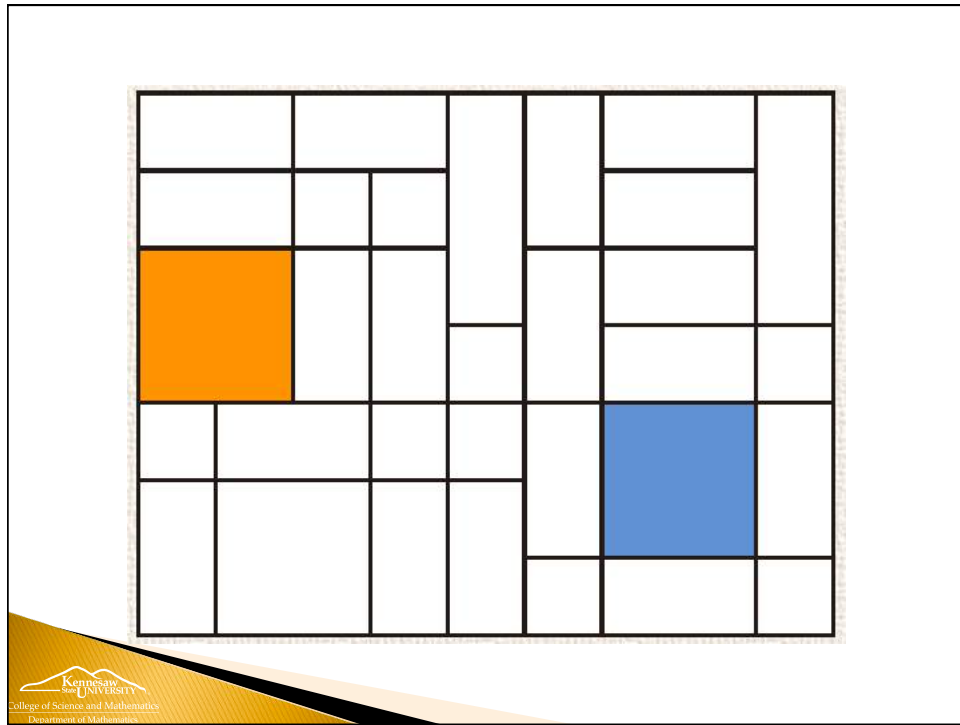
## Capture More Territory

Let's play a game using the picture below.

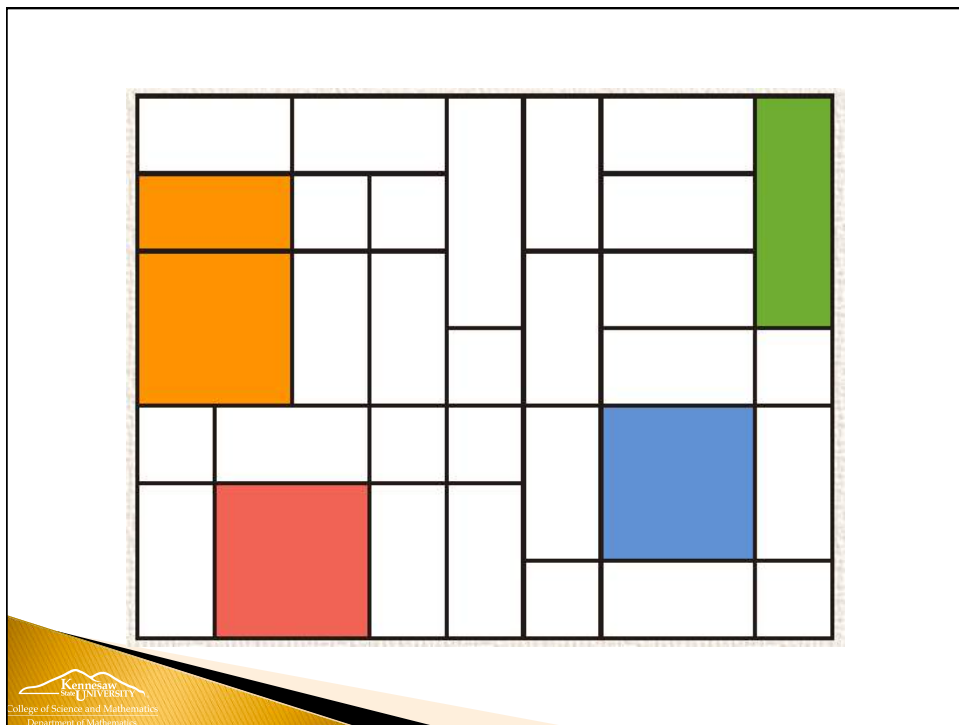
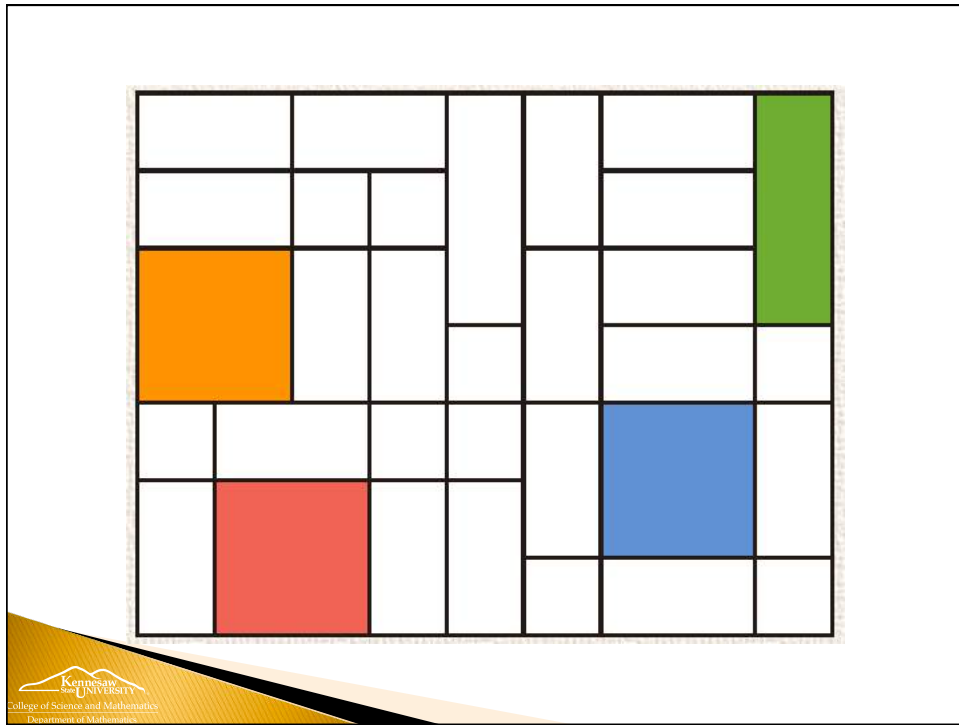
- Play Rock-Paper-Scissors. The person who wins will shade in a rectangle and it becomes his or her territory. The first time you win, you can pick any rectangle. After that, you can only capture a rectangle that is adjacent to your territory.
- The person whose territory is the largest wins the game.

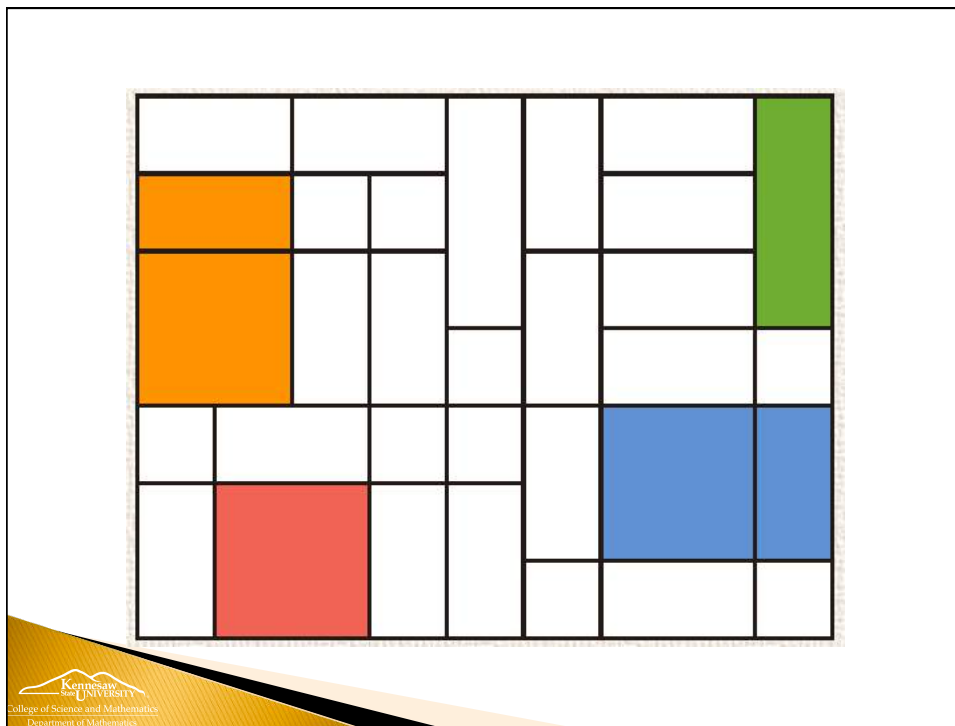












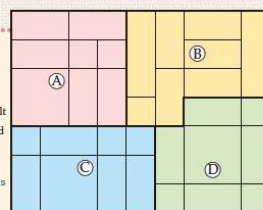
Let's compare!

# 11

How to Measure and Express Area  
**Let's Investigate the Size of Space**

The picture on the right shows the result after 4 people played the game.

Which territory has the most space?



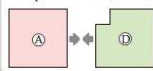
## 1 How to Express an Amount of Space

1 Compare the amount of space in the four territories, A–D.

★ Explain the ideas of each of the 3 students. 🧠 Use the cards on page 129 to compare.

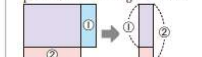
Mae

I compared territories A and D.



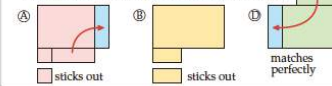
Shay

I compared territories B and C. I put them on top of each other, and the two parts that were sticking out were ...



Yumi

I put territories A, B, and D on top of the rectangle C.

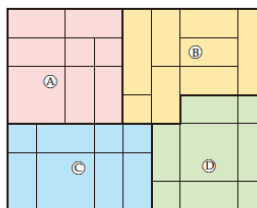
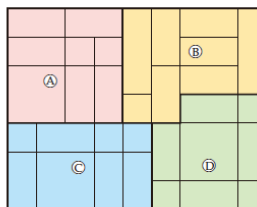
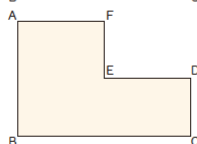
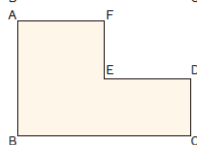
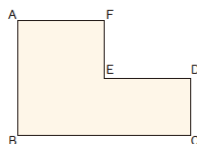
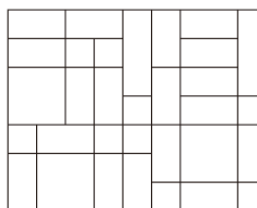
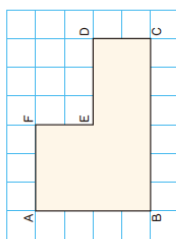


🔍 Let's investigate how to express the amount of space using numbers. 🧠 It's tedious to overlap two shapes at a time, isn't it? I wonder if there is a way to compare the shapes without overlapping them. **819**

p. 129

Used on page 25

Used on pages 18 and 19



# Direct Comparison

1 Compare the amount of space in the four territories, A~D.

★ Explain the ideas of each of the 3 students. Use the cards on page 129 to compare.

Miho  
I compared territories A and D.

Shinj  
I compared territories A and C. I put them on top of each other, and the two parts that were sticking out were ...

Yumi  
I put territories A, B, and D on top of the rectangle C.

? Let's investigate how to express the amount of space using numbers. It's tedious to overlap two shapes at a time, isn't it? I wonder if there is a way to compare the shapes without overlapping them. B19

# Non-Standard Units



Kaori

I expressed the amount of space for A and C using the boxes that are the same size.

(big)



{ A...1 piece  
C...1 piece



{ A...5 pieces  
C...4 pieces

(small)



{ A...2 pieces  
C...3 pieces

The total number of boxes, 8, is the same, but ...

I wonder if there is a unit even for an amount of space.



Hiroki

# Non-Standard Units



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(small)



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The total number of boxes, 8, is the same, but ...

Using these numbers, compare A and C.

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Hiroki

# Introduction of Standard Unit

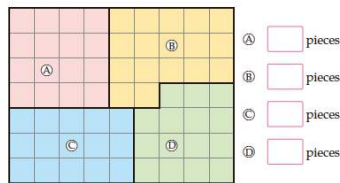
I expressed the amount of space for ㉔ and ㉕ using the boxes that are the same size.  
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2 Which of the four territories ㉔~㉗ has more space, and how much more?

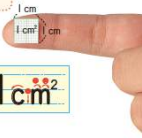
Let's think about how to express the amount of space.

As shown below, we subdivided the territories into squares with 1 cm sides. How many 1 cm squares are in each territory?



- Which territory has the most space?
- By how many 1 cm squares is territory ㉕ larger than territory ㉖?

The amount of space is called the **area**. We can express the area by the number of 1 cm squares that can be placed inside.

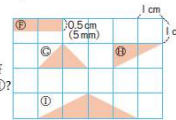


The area of a square with 1 cm sides is **1 square centimeter**, and it is written as  $1 \text{ cm}^2$ .

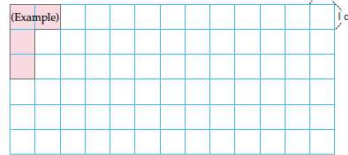
"Square centimeter" is a unit for area.

In  $\text{cm}^2$ , what is the area of each territory, ㉔~㉗?

- The area of shape ㉕ is  $1 \text{ cm}^2$ . Explain why.
- In  $\text{cm}^2$  what is the area of each figure, ㉖, ㉗, and ㉘?



Draw different shapes with an area of  $4 \text{ cm}^2$ .



# Introduction of Standard Unit

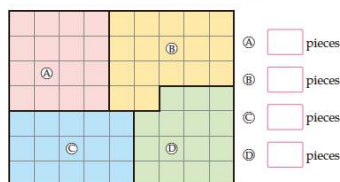
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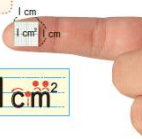
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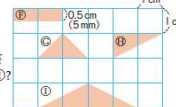


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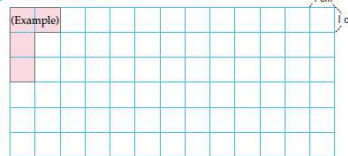
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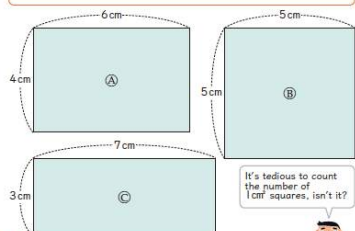
Draw different shapes with an area of  $4 \text{ cm}^2$ .



# From Counting to Calculating

## 2 Area of Rectangles and Squares

1 Think about how to find the areas of the rectangles and square below.

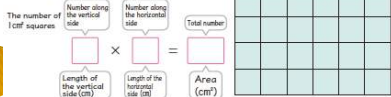


It's tedious to count the number of 1 cm<sup>2</sup> squares, isn't it?

Let's think about how to calculate the area.

How many 1 cm<sup>2</sup> squares can be placed vertically in rectangle A?

Calculate the number of 1 cm<sup>2</sup> squares that can be placed inside rectangle B. In cm<sup>2</sup>, what is the area?



The number of 1 cm<sup>2</sup> squares that can be placed along the vertical and the horizontal sides are the same as the numbers for the lengths of those sides, aren't they?

Calculate the areas of square B and rectangle C.

B  $\square \times \square = \square \square \text{ cm}^2$

C  $\square \times \square = \square \square \text{ cm}^2$

The area of rectangles and squares can be calculated as follows.

To calculate the area of rectangles and squares.

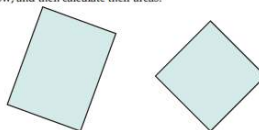
- measure the lengths of adjacent sides
- multiply the two numbers that express the lengths of the sides

Math sentences like the ones on the right are called **formulas**. Here we see the formulas for the areas of squares and rectangles.

**Area of Rectangle**  
= Length (Length of Vertical Side)  
× Width (Length of Horizontal Side)  
= Width (Length of Horizontal Side)  
× Length (Length of Vertical Side)  
**Area of Squares = Side × Side**

- 1 What are the areas of the following rectangle and square in cm<sup>2</sup>.  
① a postcard with a vertical side of 14cm and a horizontal side of 9cm  
② a piece of origami paper, with sides that are each are 15cm long

- 2 Measure the lengths of the sides of the rectangle and the square below, and then calculate their areas.



# From Counting to Calculating

How many unit squares?

1	2	3
4	5	6
7	8	9
10	11	12

1	2	3
2		
3		
4		

$4 \times 3 = 12$

1	2	3
2		
3		
4		

Number of rows of unit squares

$4 \times 3 = 12$

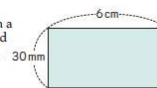
Number of unit squares in each row

## Deepening the understanding of the formula

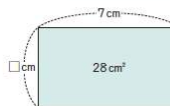
- ▶ We are counting unit squares.
- ▶ Reverse thinking.
- ▶ Perimeter and area: equal perimeter does not necessarily mean equal area.

3 Find the area of the rectangle with a vertical side that is 30mm long and a horizontal side that is 6cm long.

To calculate the area, we must use the same unit for the lengths of sides, right?



4 In order to draw a rectangle with an area of 28cm<sup>2</sup> and a horizontal side of 7cm, how long should the vertical side be?



$$\square \times 7 = 28$$

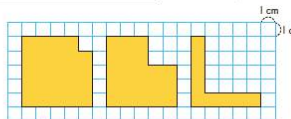
5 Figures (A), (B), and (C) on page 22 are rectangles and a square that all have perimeters of 20cm. In the table below, summarize the areas and the length of vertical and horizontal sides of the rectangles and square with the perimeters of 20cm.

	Vertical length	Horizontal length	Area	Perimeter
(A)	4	6		
(B)	5	5		
(C)	3	7		
	2			
	1			

Even though the perimeters are the same, the areas are...



These shapes also have perimeters of 20cm.



024

## Area of rectilinear figures

- ▶ **Rectilinear figure.** A polygon all angles of which are right angles. (CCSS p. 86)
- ▶ 3.MD.7.d  
Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

**Methods for finding area**

2 Find the area of the shape on the right.

**Who and her classmates are explaining their friends' ideas.**

**Hiroki:**  $4 \times 6 - 2 \times 2 = 24 - 4 = 20$   
Answer: 20 cm<sup>2</sup>

**Takumi:**  $4 \times 6 - 2 \times 2 = 24 - 4 = 20$   
Answer: 20 cm<sup>2</sup>

**Yumi:**  $4 \times (6 + 2) + 2 = 4 \times 8 + 2 = 36 + 2 = 38$   
Answer: 38 cm<sup>2</sup>

**Let's think about it by itself!**  
Is there anything you have learned so far that you can use?

**Write down your ideas!**  
Write down in a way that other people can understand.

**Let's remember!**  
What did you learn in today's lesson?

**Let's check!**  
Try doing problems using other methods to be sure today.

**Calculate the area of the shape below in many different ways.**

**What are some similarities and differences between these and your idea?**

**Discuss it with your classmates!**  
Did you find good points that your friends made?

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# Problem

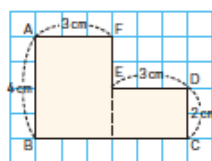
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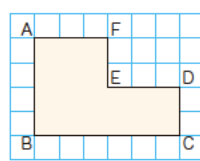
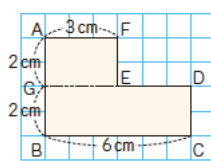
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College of Science and Mathematics  
Department of Mathematics



## Possible Solutions



$$4 \times 3 + 2$$

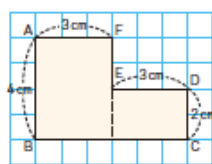


$$4 \times 6 - 2 \times 3 = 24 - 6$$

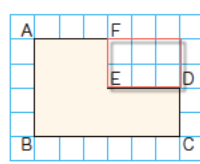
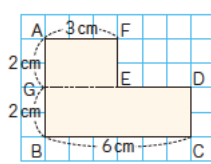
$$= 18$$

Answer  $18\text{cm}^2$

## Possible Solutions



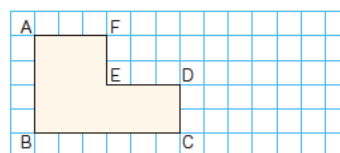
$$4 \times 3 + 2$$



$$4 \times 6 - 2 \times 3 = 24 - 6$$

$$= 18$$

Answer  $18\text{cm}^2$

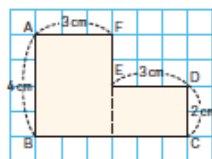


$$4 \times (6 + 3) + 2 = 4 \times 9 + 2$$

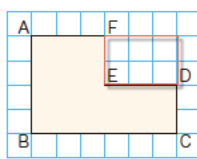
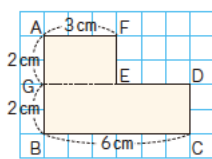
$$= 36 + 2$$

$$= 38 \quad \text{Answer } 18\text{cm}^2$$

## Possible Solutions



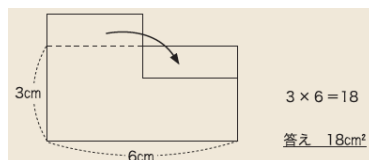
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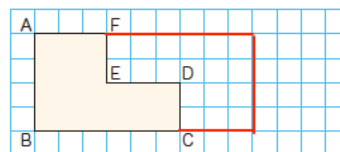
$$= 18$$

Answer  $18\text{cm}^2$



$$3 \times 6 = 18$$

答え  $18\text{cm}^2$

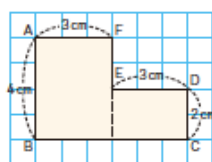


$$4 \times (6 + 3) \div 2 = 4 \times 9 \div 2$$

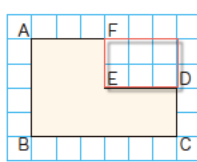
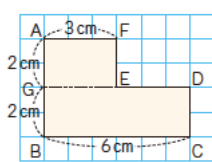
$$= 36 \div 2$$

$$= 18 \quad \text{Answer } 18\text{cm}^2$$

We can find the area of unfamiliar shapes by making use of familiar shapes.



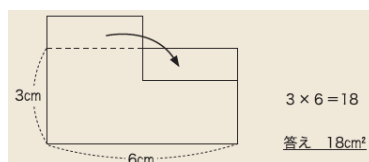
$$4 \times 3 + 2$$



$$4 \times 6 - 2 \times 3 = 24 - 6$$

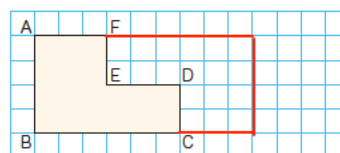
$$= 18$$

Answer  $18\text{cm}^2$



$$3 \times 6 = 18$$

答え  $18\text{cm}^2$



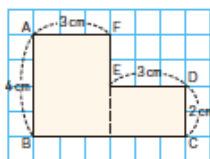
$$4 \times (6 + 3) \div 2 = 4 \times 9 \div 2$$

$$= 36 \div 2$$

$$= 18 \quad \text{Answer } 18\text{cm}^2$$

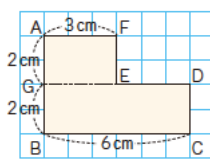
## Strategies to make use of familiar shapes

Sub-divide, find the sum



$$4 \times 3 + 2 \times 3$$

Add on then subtract

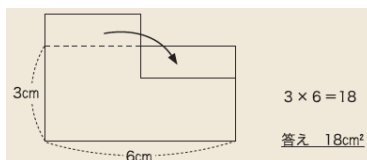


$$4 \times 6 - 2 \times 3 = 24 - 6$$

$$= 18$$

Answer  $18\text{cm}^2$

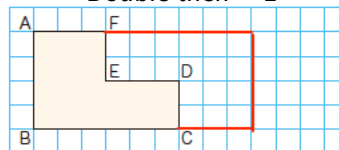
Cut and re-arrange



$$3 \times 6 = 18$$

答え  $18\text{cm}^2$

Double then  $\div 2$



$$4 \times (6 + 3) \div 2 = 4 \times 9 \div 2$$

$$= 36 \div 2$$

$$= 18 \quad \text{Answer } 18\text{cm}^2$$

## Grade 5 Unit 11

# 11

Area of Quadrilaterals and Triangles

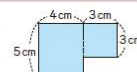
Let's Think about How to Find Area



Since ① is a square and ② is a rectangle, we can find their areas using a formula can't we?



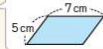
We can find the area of ③ by splitting it into a rectangle and a square.



④ is a parallelogram. Since there are slanted sides, counting the number of  $1\text{cm}^2$  squares is ...

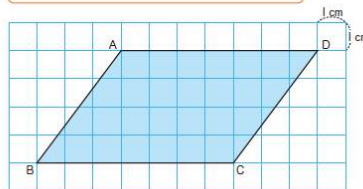


I wonder if we can find the area of a parallelogram by multiplying the lengths of two adjacent sides.



### 1 How to Find the Area of Parallelograms

1 Think about how to find the area of parallelogram ABCD below.



# Deriving the formula for parallelogram

★ Explain the two students' ideas below.

**Kaori**

Kaori moved triangle ECD to change parallelogram ABCD into rectangle FBCE to find the area.

**Shinji**

Why did Kaori change the shape into a rectangle?

**Hiroki**

Hiroki moved quadrilateral GKCD to change parallelogram ABCD into ...

**Yumi**

★ The area of this parallelogram is equal to the area of a rectangle with what dimensions?

★ What is the area of this parallelogram?

The area of a parallelogram can be found if by changing it into a rectangle.

Even though the shapes are changed, the area stays the same, doesn't it?

Using the two students' ideas above, find the area of the parallelogram on the right.

2 Based on Kaori's idea on the previous page, calculate the area of parallelogram ABCD on the right.

Let's come up with a formula to find the area of a parallelogram.

★ The area of the parallelogram above is equal to the area of a rectangle with what dimensions?  
Which parts of the parallelogram are equal to the length and the width of this rectangle?

Draw the part that is the same length as the vertical side of the rectangle in the figure above.

In the parallelogram on the right, if we make side BC as the **base**, the length of the segments that are perpendicular to the base such as segment EC is called the **height** of the parallelogram.

If we consider side AB as the base, the height will be the length of the segments such as those shown on the right.

★ Calculate the area of the parallelogram above by considering side BC as the base.

The area of a parallelogram can be calculated using the following formula.

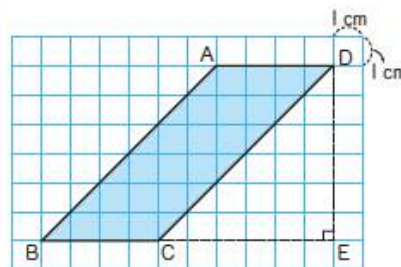
**Area of parallelogram = Base × Height**

# How about this parallelogram?

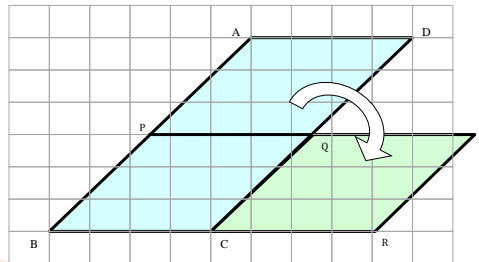
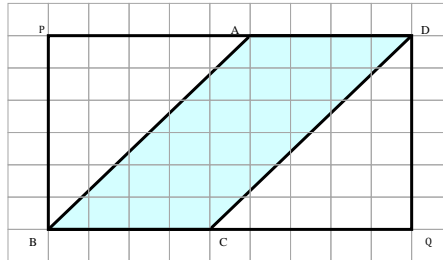
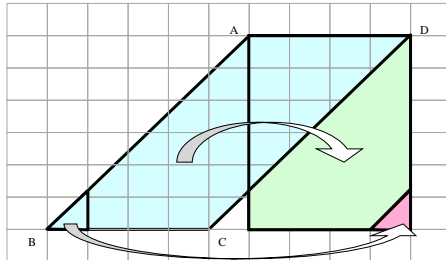
3 Think about how to find the area of parallelogram ABCD on the right when side BC is considered as the base.

The height is ...

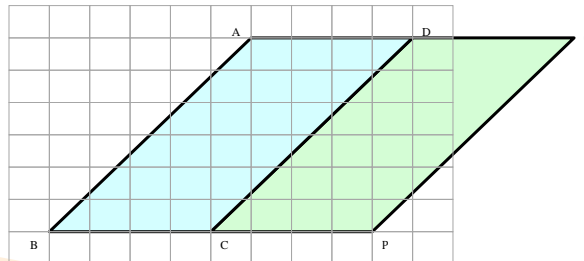
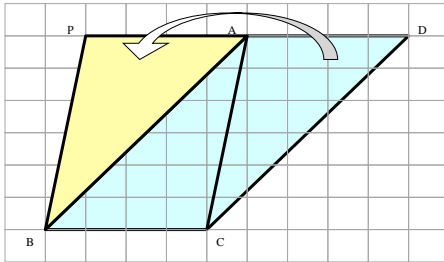
**Kaori**



## Patterson School #2

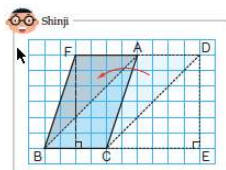


## More from Patterson School #2



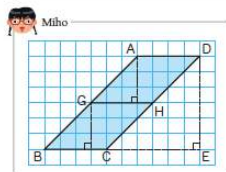
- Any side of a parallelogram can serve as the base.
- The height is the distance between the parallel lines that contain the base and its opposite side – therefore the height can be found outside of the parallelogram.

★ Explain the two students' ideas using the diagrams.



Yumi

Shinji moved triangle ACD to make parallelogram FBCE with a height on the inside like before.

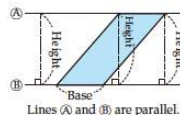


Takumi

Miho split parallelogram ABCD into two parallelograms AGHD and GBCH with a height on the inside like before.

★ Based on the two students' ideas, calculate the area of parallelogram ABCD.

As shown on the right, sometimes the height of a parallelogram can be found outside of the figure.



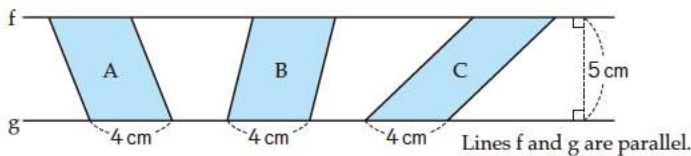
★ If BC is the base, what is the height in cm?

★ Calculate the area to see if the formula on page 31 still applies. Use BC as the base and the appropriate height.

B33

## Deepening the understanding

4 Find the area of parallelograms A, B, and C below.



★ Explain why the area of parallelograms A, B, and C are equal.

Even if parallelograms are shaped differently, if the lengths of their bases and heights are equal, their areas will also be equal.

Summary

# Triangles

Three students are reviewing what they have studied about area so far.

**1** No matter what its shape, the area of a parallelogram can be calculated if we know the base and the height.

**2** I wonder if we can find the area of other geometric figures we have studied.

I want to find the area of triangles.

**1** Explain the three students' ideas below using diagrams and math sentences.

**Yusef:**  $7 \times 4 = 28$

**Maya:** You used the fact that you can make a parallelogram using two copies of triangle ABC, and ...

**Takumi:**  $4 \times 7 = 28$

**Kami:** Takumi made rectangle EBGD using 2 copies each of triangle ABF and triangle AFC, and ...

**Mika:** Mika made rectangle LKMN by moving triangle ANL and triangle AKC, and ...

**2** Based on Yusef's idea on the previous page, calculate the area of triangle ABC on the right.

**2** Let's come up with a formula for calculating the area of a triangle.

The area of the triangle above is a half the area of a parallelogram. What is the base and the height of the parallelogram? Which parts of the triangle correspond to the base and the height of the triangle?

In triangle ABC on the right, the length of segment AD, which is perpendicular to the base BC, is called the **height** of the triangle.

If side AB is considered the base, then the height is the length of the segment on the right.

Calculate the area of the triangle above using BC as the base.

The area of a triangle can be calculated using the following formula:

**Area of triangle = Base  $\times$  Height  $\div$  2**

I wonder if we can make this formula about Takumi's or Mika's ideas, too?

This means that the Area of triangle = Area of parallelogram  $\div$  2, doesn't it?

I wonder if we can change the shape into something that we already know how to find the area of.

What is the area of triangle ABC?

You can find the area of a triangle by changing its shape to a rectangle or a parallelogram.

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# Triangles

Calculate the area of the following triangles.

**1**  $6\text{ cm}$ ,  $9\text{ cm}$

**2**  $5\text{ cm}$ ,  $6\text{ cm}$ ,  $6\text{ cm}$

**3**  $6\text{ m}$ ,  $8\text{ m}$ ,  $10\text{ m}$

**4**  $1\text{ cm}$ ,  $3\text{ cm}$

**1** Explain the following two students' ideas using the diagrams.

**Kasri:** Kasri used the fact that two triangles ABC can be put together to make parallelogram EBGA, then ...

**Hiroki:** Hiroki subtracted the area of triangle ACD from the area of triangle ABD to ...

Why did Hiroki think about using triangles ABD and ACD?

**2** Based on these two students' ideas, find the area of triangle ABC.

As shown on the right, the height of a triangle can be found outside of the triangle.

What is the height of the triangle if we consider BC as the base?

Check to see if we can use the formula on page 27 by substituting the length of BC as the base and the height found above.

Measure the necessary lengths and calculate the area of the triangle on the left.

**3** Think about how to find the area of triangle ABC on the right when we consider BC as the base.

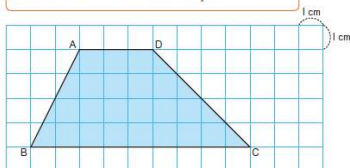
The height is ...

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# Trapezoids and Rhombuses

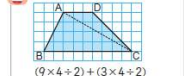
## 3 How to Find the Area of Various Quadrilaterals

1 Think about how to find the area of trapezoid ABCD.



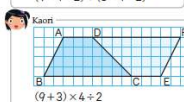
★ Explain the following 3 students' ideas using diagrams and math sentences.

Takumi



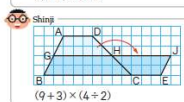
Since we can split trapezoid ABCD into triangles ABC and ADC using a diagonal ...

Yumi



Since we can form parallelogram ABFE by combining two of trapezoid ABCD ...

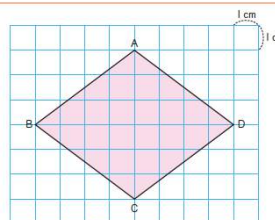
Itiroki



Since we can make parallelogram GBEJ by moving trapezoid AGHD ...

Miho

3 Think about how to find the area of rhombus ABCD below.



★ Explain the following three students' ideas using diagrams and math sentences.

Kaori

By splitting it into 2 triangles ...



$$8 \times 3 + 2 \times 2$$

Itiroki

By thinking of it as a half of a rectangle ...



$$6 \times 8 \div 2$$

Miho

By changing it into a rectangle ...



$$8 \times (6 \div 2)$$

All three of them are changing the rhombus into a shape that we can calculate the area for, aren't they?

★ What is the area of rhombus ABCD?

841

844

## Big Ideas

- ▶ Formulas are useful, but
- ▶ ... thinking about ways to calculate the area of unfamiliar shapes, or deriving formulas, is the focus of instruction.
- ▶ **BIG IDEA**

We can find the area of an unfamiliar shape by making use of familiar shapes.



Thank you.



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