

## Teaching through Problem Solving

Tom McDougal  
Lesson Study Alliance

## CCSSM SMP1

- Students will understand problems and persevere in solving them.

## NCTM view of problem solving

- Problem solving means **engaging in a task for which the solution is not known in advance.**
- Good problems give students the chance to solidify and extend their knowledge and to **stimulate new learning.** Most **mathematical concepts can be introduced through problems based on familiar experiences coming from students' lives or from mathematical contexts.**
- Students need to develop a range of strategies for solving problems, **such as using diagrams**, looking for patterns, or trying special values or cases.

## Example of "problem solving" worksheet

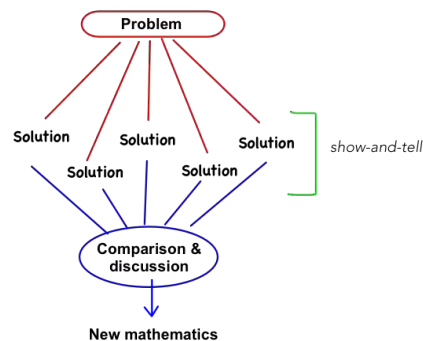
**You are selling ice cream from a cart. You sell ice cream bars for \$0.75 per bar. Your cost for the ice cream is \$0.30 per bar, and your cost for the rental of the cart is \$50.**

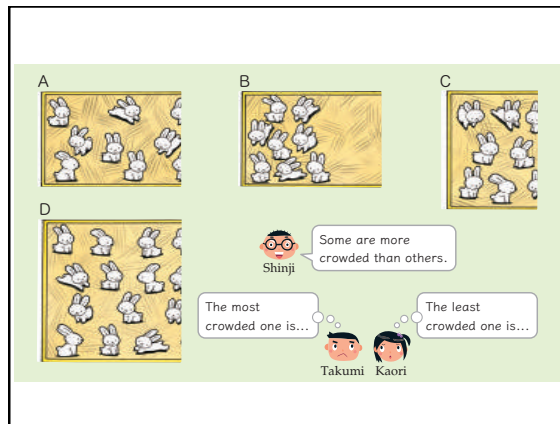
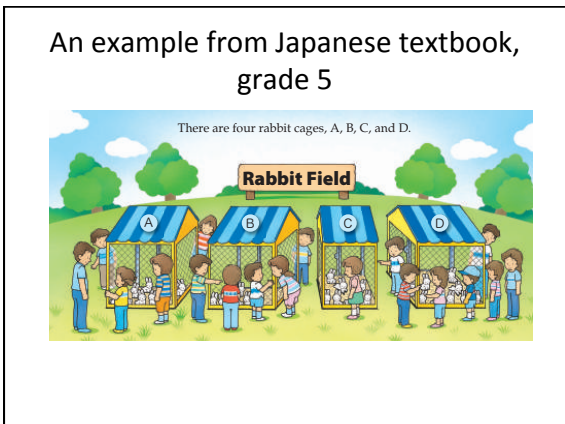
- In a formula, express your total cost  $C$  as a function of the number  $n$  of ice cream bars sold. On graph paper, graph  $C$  leaving room for negative values on the y-axis.
- Express the revenue  $R$  generated by the sale of ice cream bars as a function of the number  $n$  sold. Graph on the same graph as in  $a$ .
- Express the profit  $P$  generated by the sale of ice cream bars as a function for the number  $n$  sold. Graph  $P$  on the same graph as in  $a$  and  $b$ .
- Find the break even point graphically and algebraically.

## What does a Common Core classroom look like?

- Level 1: Teachers can tell students important basic ideas of mathematics such as facts, concepts, and procedures.
- Level 2: Teachers can explain the meanings and reasons of the important basic ideas of mathematics in order for students to understand them.
- Level 3: Teachers can provide students opportunities to understand these basic ideas, and support their learning so that the students become independent learners.

## Teaching through Problem Solving





**Area of Cage and Number of Rabbits**

	Area (m <sup>2</sup> )	Number of rabbits
A	6	9
B	6	8
C	5	8
D	9	15

If the areas are the same or the numbers of rabbits are the same, it is easy to compare.

What about A vs C?

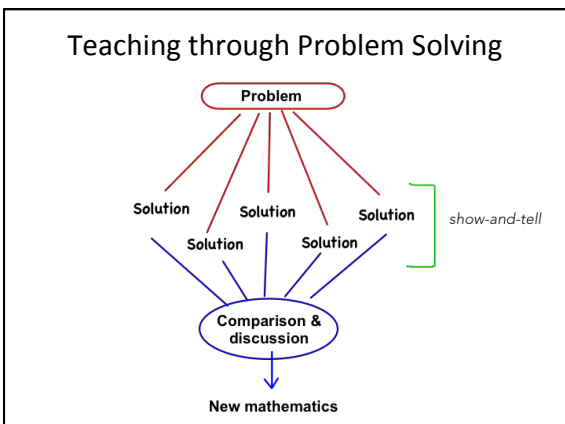
**Compare A vs C**

**Area of Cage and Number of Rabbits**

	Area (m <sup>2</sup> )	Number of rabbits
A	6	9
B	6	8
C	5	8
D	9	15

- Student 1:  
A:  $9 \times 5 = 45$  (rabbits per 30 m<sup>2</sup>)  
C:  $8 \times 6 = 48$
- Student 2:  
A:  $9 \div 6 = 1.5$  (rabbits per 1 m<sup>2</sup>)  
C:  $8 \div 5 = 1.6$
- Student 3:  
A:  $6 \div 9 = .66\dots$  (m<sup>2</sup> per 1 rabbit)  
C:  $5 \div 8 = .625$

New mathematics: You can divide to get "per unit quantities", which make it easy to compare.



**Jones research lesson**

- Grade 11 students using IMP
- Understand derivative of function as slope of tangent line of graph
- Determine derivative qualitatively (pos, neg, zero)
- Have gone "backwards", sketching graph of fn given info about where derivative is pos, neg, zero

- Lesson goal: Strengthen students understanding of the relationship between the graph of a function and the graph of its derivative by having students work “backwards”—determining the shape of the graph of a function from the graph of the derivative.

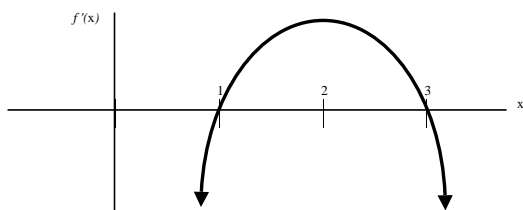
## Warm-up

Sketch a graph of a function  $f(x)$  that fits all of these conditions.

$x$	$f'(x)$
1	Negative
2	0
3	Positive

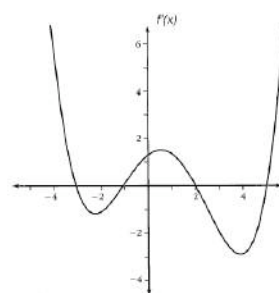
## Main problem

Sketch a graph of a function  $f(x)$  that fits all of these conditions.



## 2<sup>nd</sup> problem (assessment)

Sketch a graph of a function  $f(x)$  that fits all of these conditions.

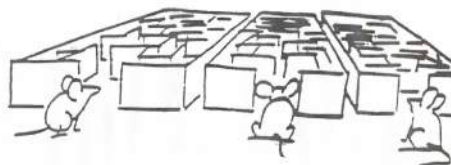


## (show videos)

[http://hrd.apec.org/index.php/Interpreting\\_the\\_Graph\\_of\\_the\\_Derivative\\_of\\_a\\_Function](http://hrd.apec.org/index.php/Interpreting_the_Graph_of_the_Derivative_of_a_Function)

## How does one learn to teach like this?

This doesn't work:



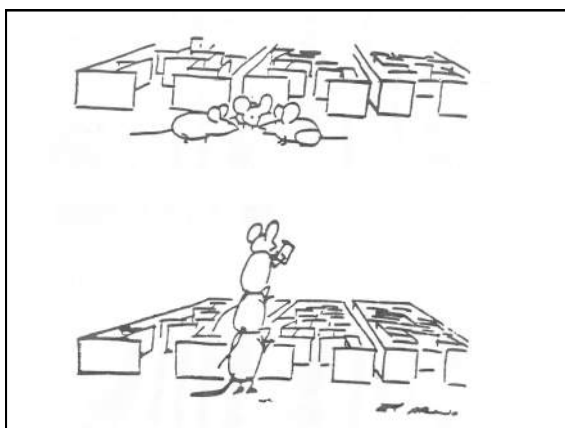
### Challenges

- Textbooks are typically written to support level 2 teaching (at best).
- Difficult to come up with suitable problems.
- Students expect level 2 teaching.
- It's hard to do!

### Use Lesson Study to overcome the challenges

Work with 2-5 colleagues:

1. Select a troublesome unit topic
2. Research the topic—look at different curricula
3. Plan a unit based on research
4. Carefully plan one lesson within the unit to be a “teaching through problem solving” lesson. An experiment!
5. One teacher teaches, the others observe: What is the impact of OUR lesson on the students?
6. After the lesson, discuss what worked or not, seek ideas for future lessons.



### Please let us help!

- Lesson Study Alliance  
773-888-3404  
Tom McDougal  
<TFMcDougal@LSAlliance.org>