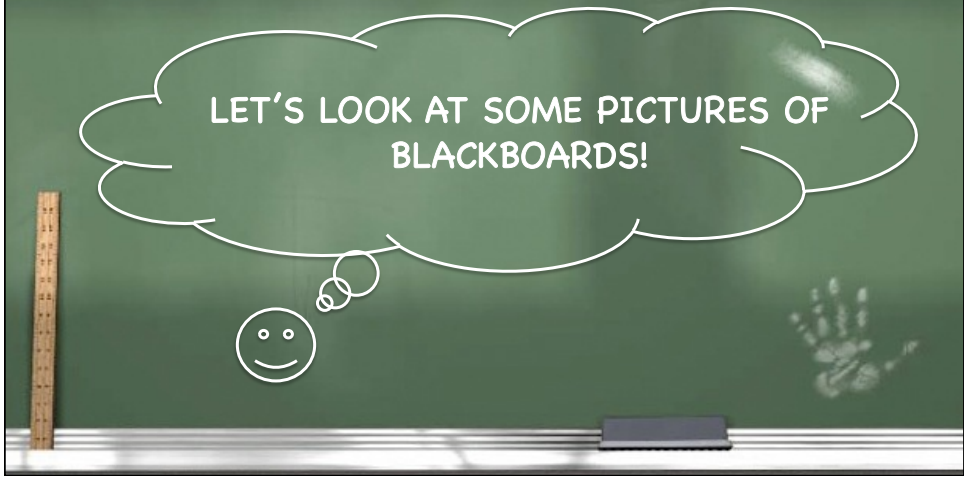
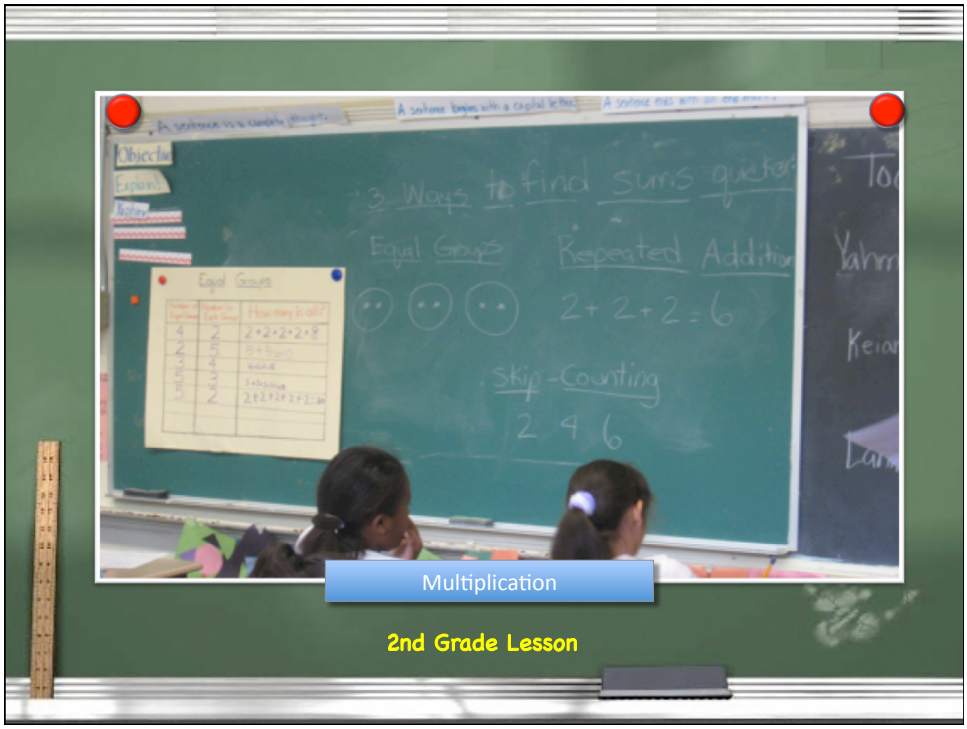
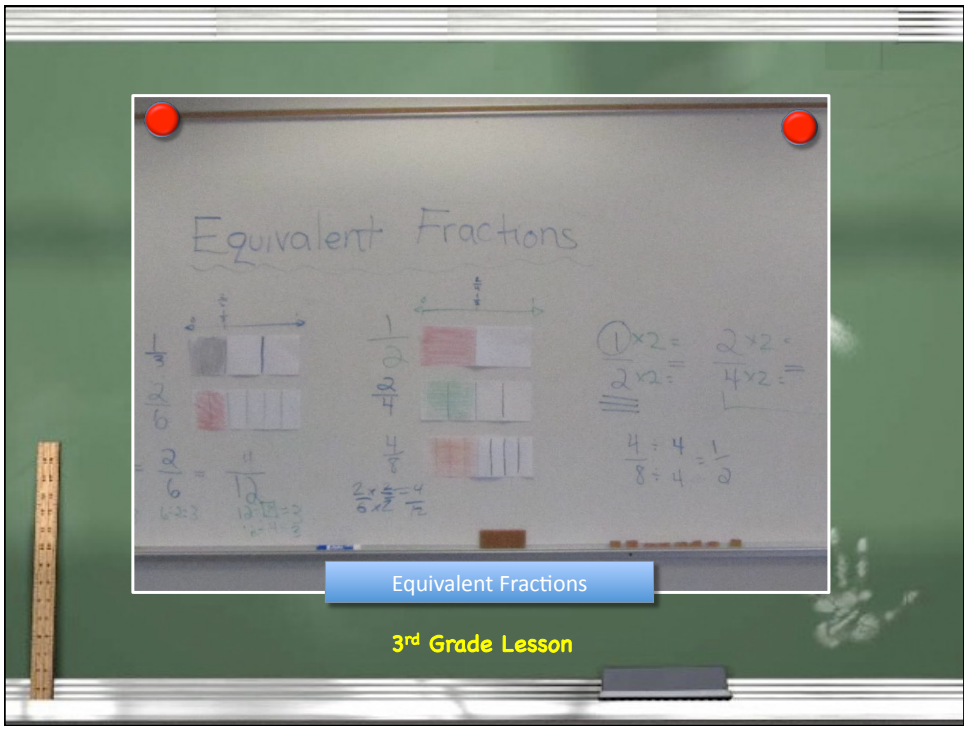
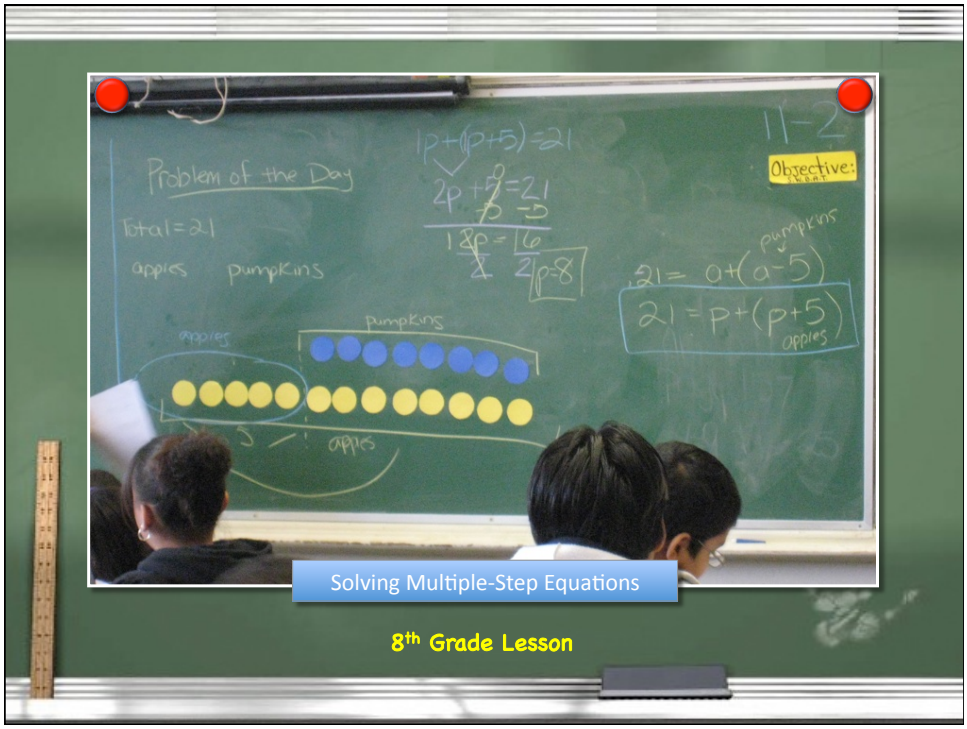
Bansho （板書）：
Using the Blackboard Effectively to Support the Mathematical Practices

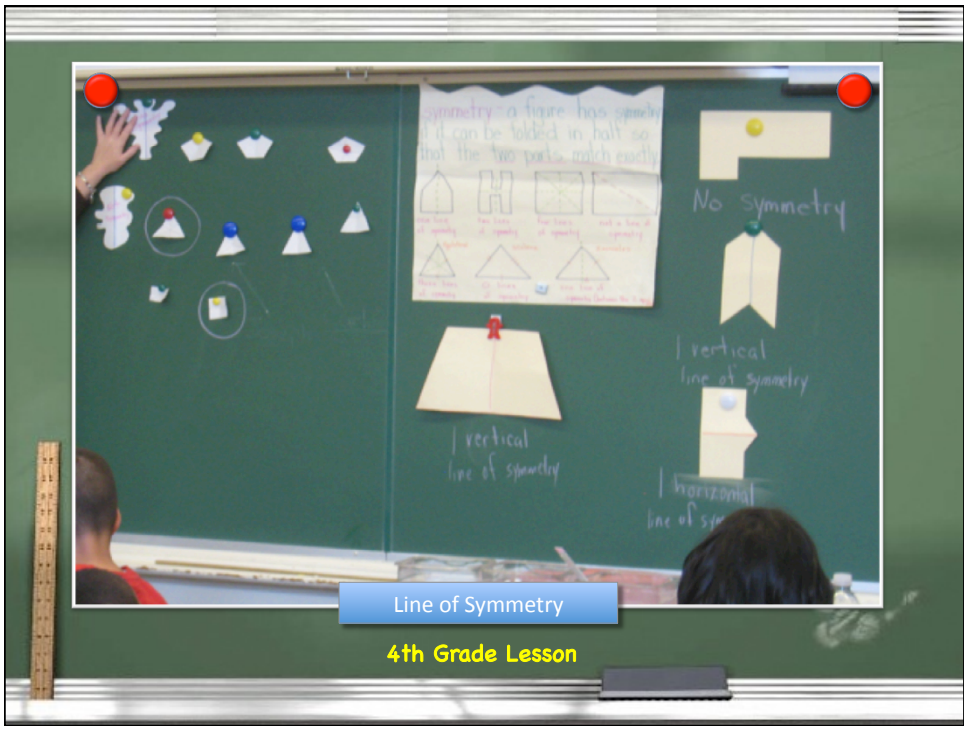
Makoto Yoshida
yoshidam@wpunj.edu
William Paterson University

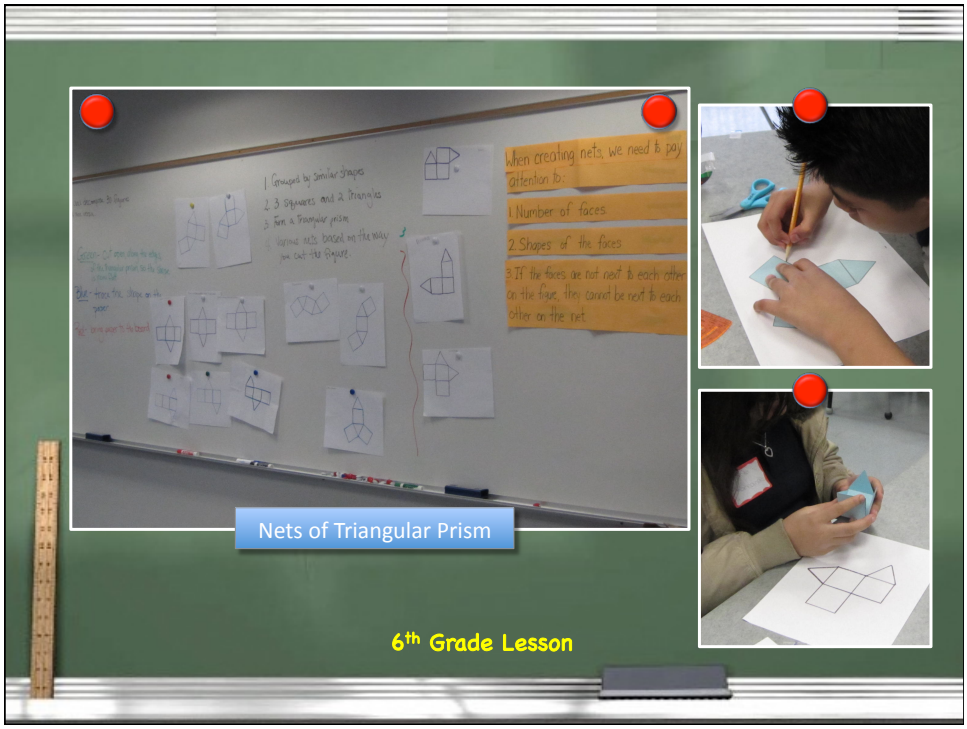


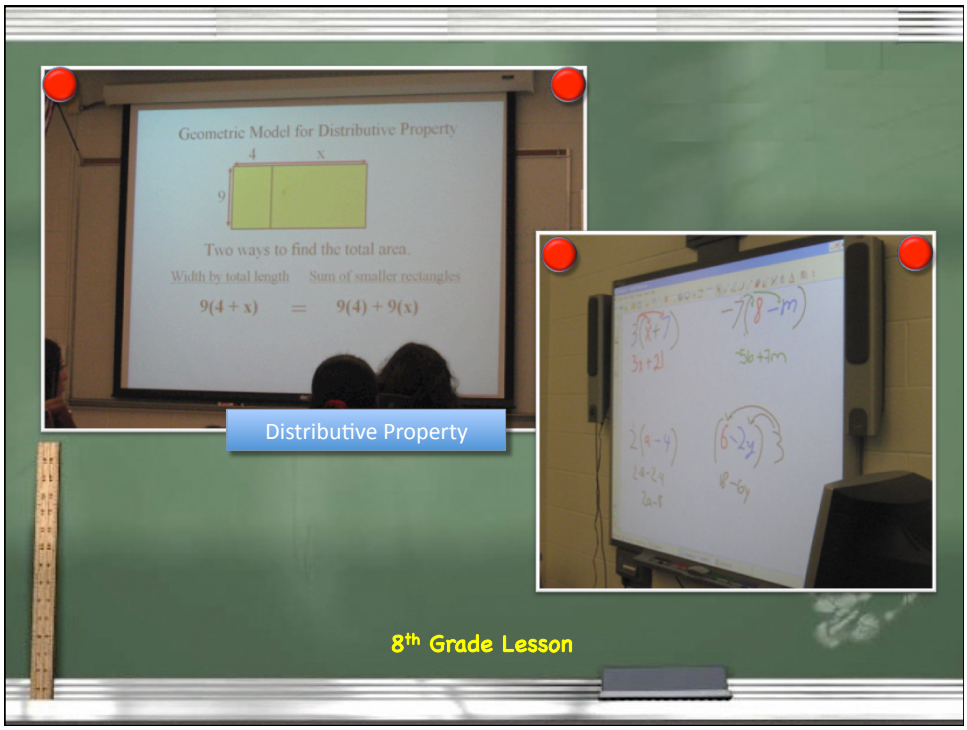

3 Ways to find sums quicker
Equal Groups
Repeated Addition
2+2+2=6
Skip-Counting
2 4 6
Multiplication
2nd Grade Lesson


Equivalent Fractions
Equivalent Fractions
3rd Grade Lesson


Problem of the Day
Total = 21
apples
pumpkins
p+(p+5)=21
2p+5=21
-5 -5
2p=16
2 2
p=8
11-2
Objective:
21 = a+(a-5)
21 = p+(p+5)
pumpkins
apples
Solving Multiple-Step Equations
8th Grade Lesson

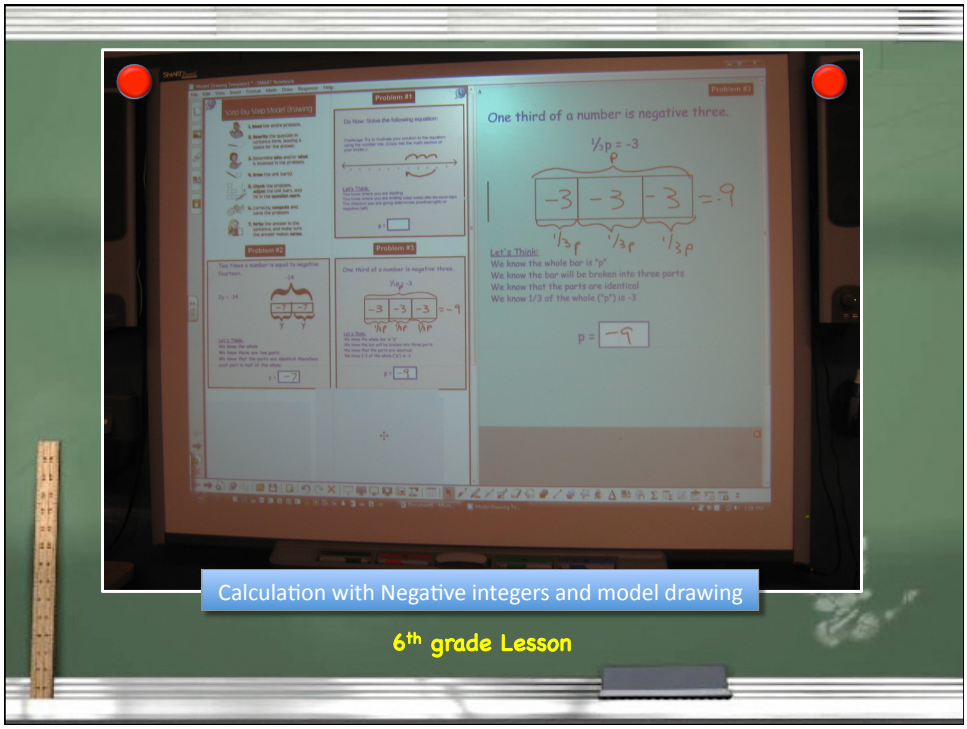

No symmetry
1 vertical line of symmetry
1 vertical line of symmetry
Line of Symmetry
4th Grade Lesson


When creating nets, we need to pay attention to:
1. Number of faces
2. Shapes of the faces
3. If the faces are not next to each other on the figure, they cannot be next to each other on the net
Nets of Triangular Prism
6th Grade Lesson


Geometric Model for Distributive Property
4
x
9
Two ways to find the total area.
Width by total length
Sum of smaller rectangles
9(4 + x) = 9(4) + 9(x)
Distributive Property
8th Grade Lesson

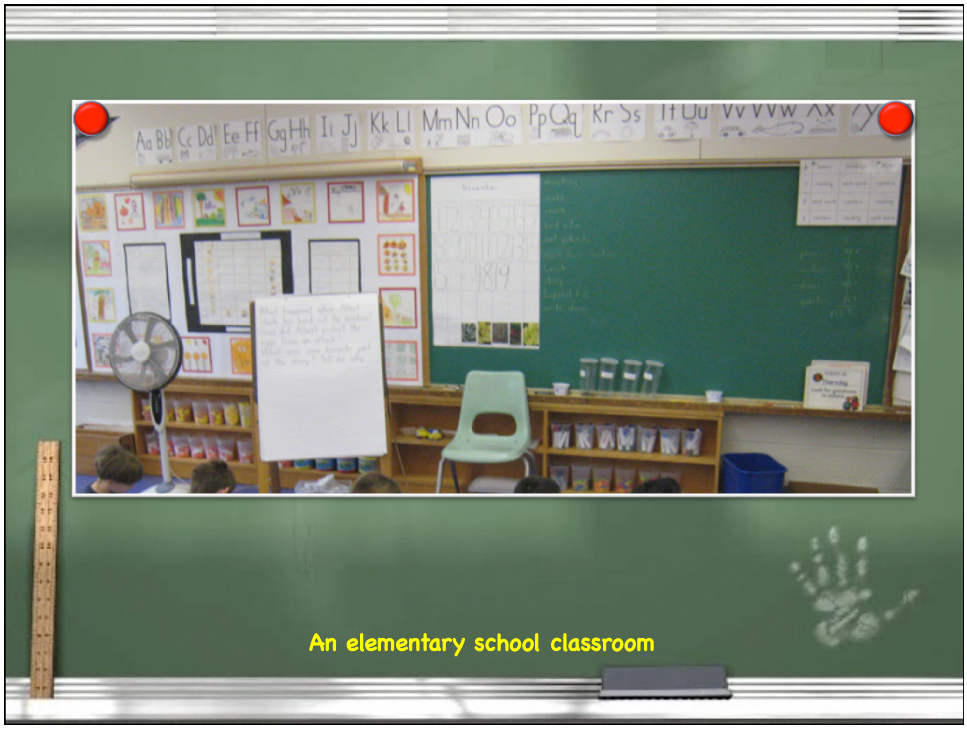
Addition

Kindergarten Lesson



Calculation with Negative integers and model drawing

6th grade Lesson


An elementary school classroom


MESSAGE
An elementary school classroom

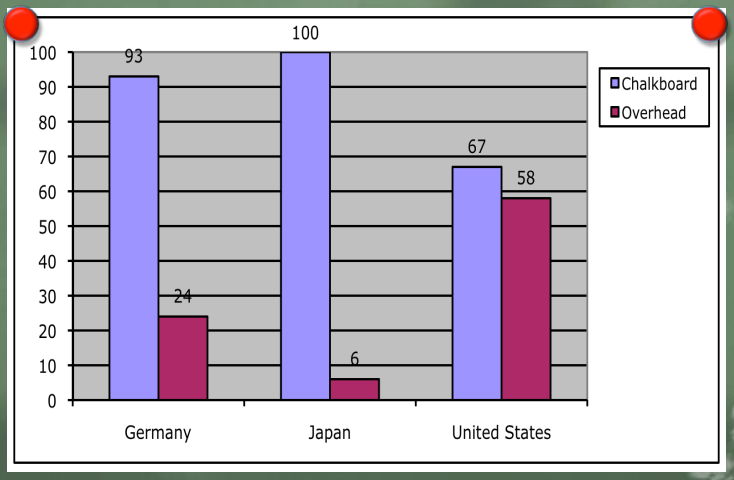
Image of Blackboards in Today's Classrooms (U.S.)

- An old instructional tool that is dusty and chalky (whiteboard is better),
- Not friendly to our skin and health,
- Not good for keeping our clothes clean
- Annoying scraping sound makes us want to scream!
- "Chalk and talk," – not a good tool
- Good for students to show their work
- OHP, Document projector, Smartboard, PowerPoints, are better

Image of Blackboards in Today's Classrooms (Japan)

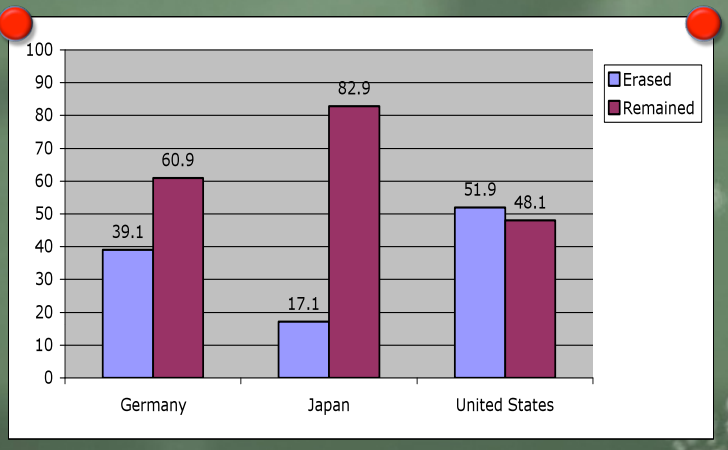
- Help to organize learning of a lesson
  - Builds coherence of lesson and providing reference points
  - Show highlight points of a lesson
  - Help to see summary of a lesson
- Help to provide opportunities for students to participate during the lesson
- Help to compare and discuss different thinking and ideas

Percentage of Use of Chalkboard and Overhead Projector in Classrooms



Source: U.S. Department of Education, National Center for Education Statistics. Third International Mathematics and Science Study, Videotape Classroom Study, 1994-95.

Percentage of Tasks, Situations, and PPDs (Principles/properties/definitions) Written on the Chalkboard that were Erased or Remained on the Chalkboard at the End of the Lesson



Source: U.S. Department of Education, National Center for Education Statistics. Third International Mathematics and Science Study, Videotape Classroom Study, 1994-95.

My Observation of Blackboard Use in Japan

- Japanese teachers rarely erase what they write on the blackboard. Everything they choose to record has a meaning and purpose, as it has been carefully planned in advance.

Source: Yoshida (1999) Lesson Study: A Case Study of A Japanese Approach to Improving Instruction Through School-Based Teacher Development

What Japanese Teachers Say about Blackboard Use

One Japanese teacher described the importance that Japanese teachers place on using the blackboard:

> "My senior teachers told me 'you should not erase what you write if you write on the blackboard and you should not write on the board if you are going to erase it.'"

Source: Yoshida (1999) Lesson Study: A Case Study of A Japanese Approach to Improving Instruction Through School-Based Teacher Development

What Japanese Teachers Say about Blackboard Use

Another Japanese teacher described it like this:

> "I try to organize the blackboard in such a way that my students and I can see and understand how the lesson progressed and what was talked about during the lesson and at the end of the lesson."

Source: Yoshida (1999) Lesson Study: A Case Study of A Japanese Approach to Improving Instruction Through School-Based Teacher Development

Use or Organization of Blackboard

板書 (Bansho) (Board-Writing)

- A technical term created by Japanese teachers
- Considered an important teaching skill
- Considered one of the necessary tools for child-centered discovery-oriented lessons or structured problem solving lesson
- Often discussed during lesson study

Mathematical Instruction

- Three Levels of Mathematical Instruction:
  - **Level 1:** Teacher knows how to perform mathematical procedures and formulas and demonstrate them to the students.
  - **Level 2:** the teacher has conceptual understanding of how and why mathematical procedures and formulas work and tries to explain this understanding to students.
  - **Level 3:** the teacher not only understands mathematical procedures and formulas and their conceptual underpinnings, but also tries to develop students' autonomy by helping them to discover and create new mathematical ideas.

(Sugiyama 2008)

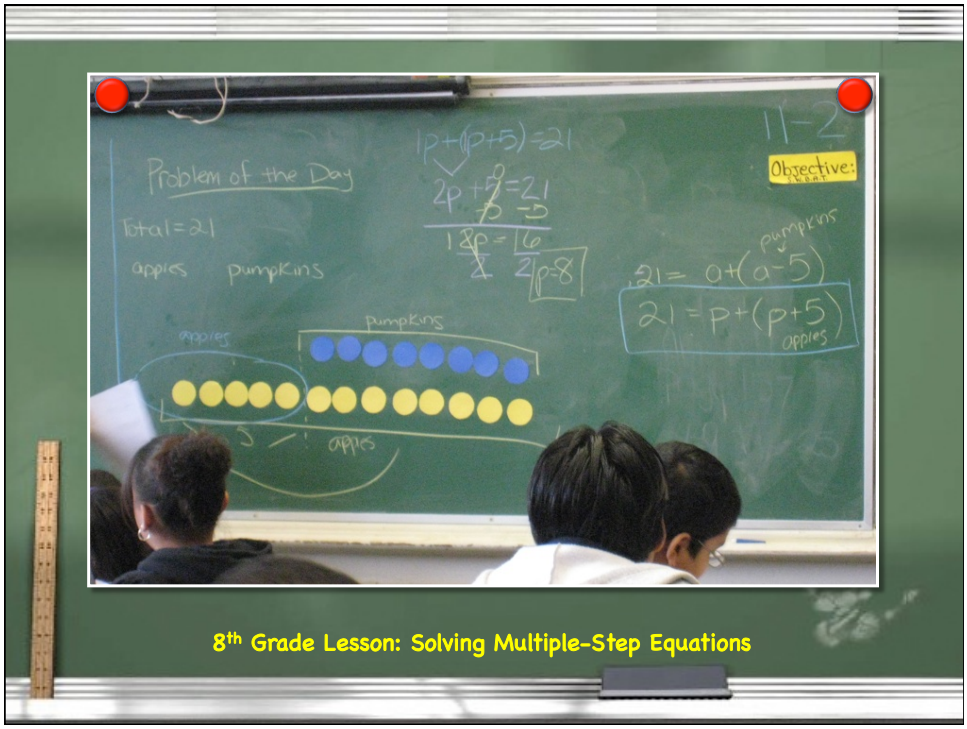
Blackboard Use: Level 1

- Recording some information on the board
- Not organized but some attempt to highlight important ideas

Level of Sophistication of Blackboard Use

Level 1

Recording some information, not organize, attempt to highlight important ideas



8th Grade Lesson: Solving Multiple-Step Equations

Blackboard Use: Level 2

1. Keep a Record of the Lesson
   - Problem
   - Questions
   - Directions to proceed activities or experiments
   - Student solutions
   - Important content ideas
2. Help students see the connections between the different parts of lesson and the progression of the lesson
   - Summary of the entire lesson
   - Coherent flow of the lesson (how we reached the conclusion)

Level of Sophistication of Blackboard Use

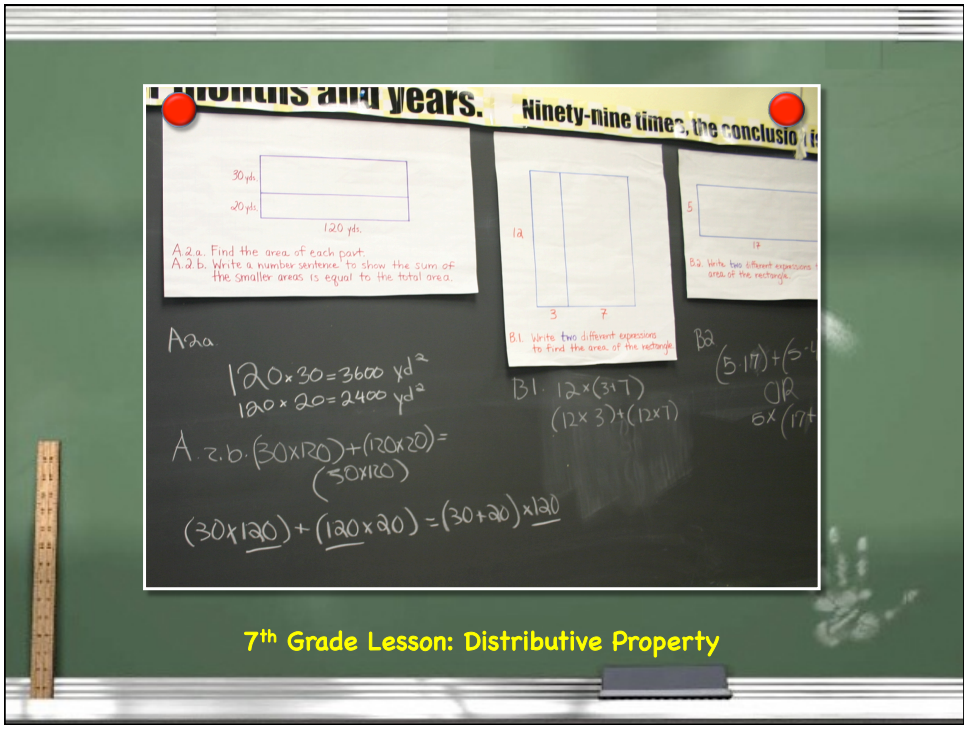
Level 2

Keep record, make connection

Level 1

Recording some information, not organize, attempt to highlight important ideas

2nd Grade Lesson: Multi-Digits Subtraction



7th Grade Lesson: Distributive Property

Blackboard Use: Level 3

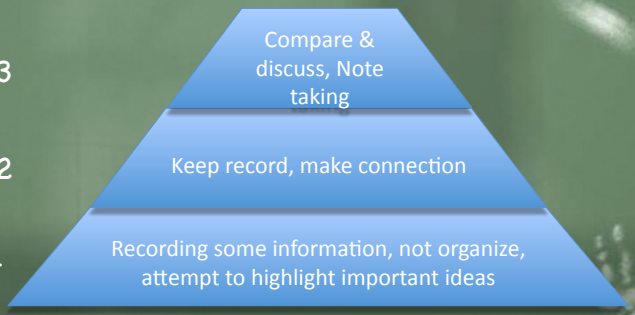
1. Help students see the connections between the different parts of the lesson and the progression of the lesson
   - Summary of the entire lesson
   - Coherent flow of the lesson (how we reached the conclusion)
2. Compare and discuss Ideas
   - Different solutions and ideas are discussed based on student presentation
   - Organizing ideas, categorizing, effectiveness, accuracy, are discussed
   - Recording student discussions and highlighting and summarizing their findings
3. Fostering students' organized note-taking skills by modeling good organization

Level of Sophistication of Blackboard Use

Level 3

Level 2

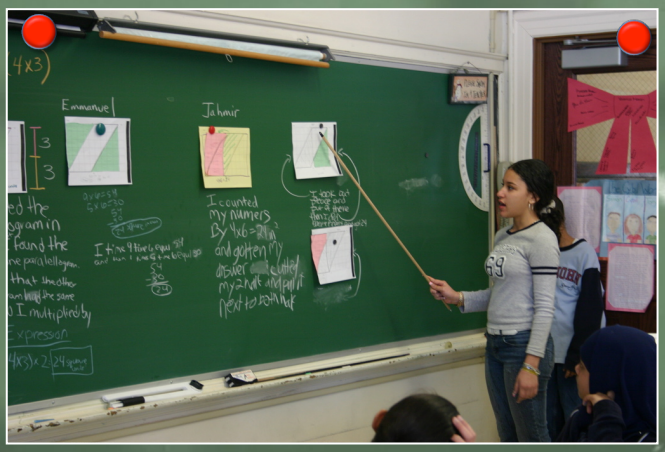
Level 1



Students Working on the Problem

6th Grade Lesson: Developing the Formula for Area of a parallelogram

A Student Presenting Her Idea



6th Grade Lesson: Developing the Formula for Area of a Triangle

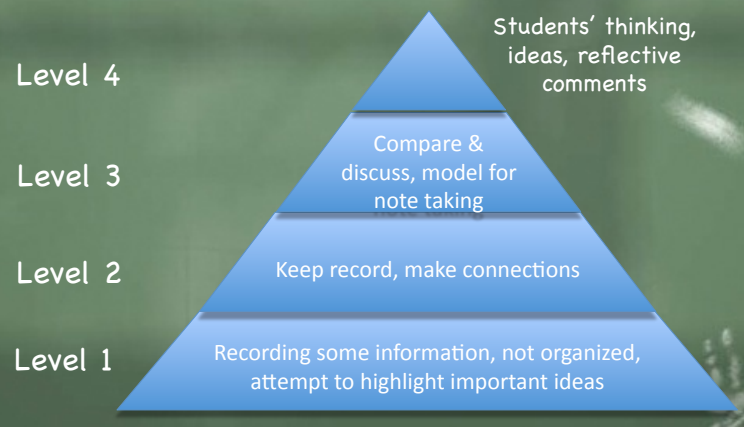
Blackboard Organization

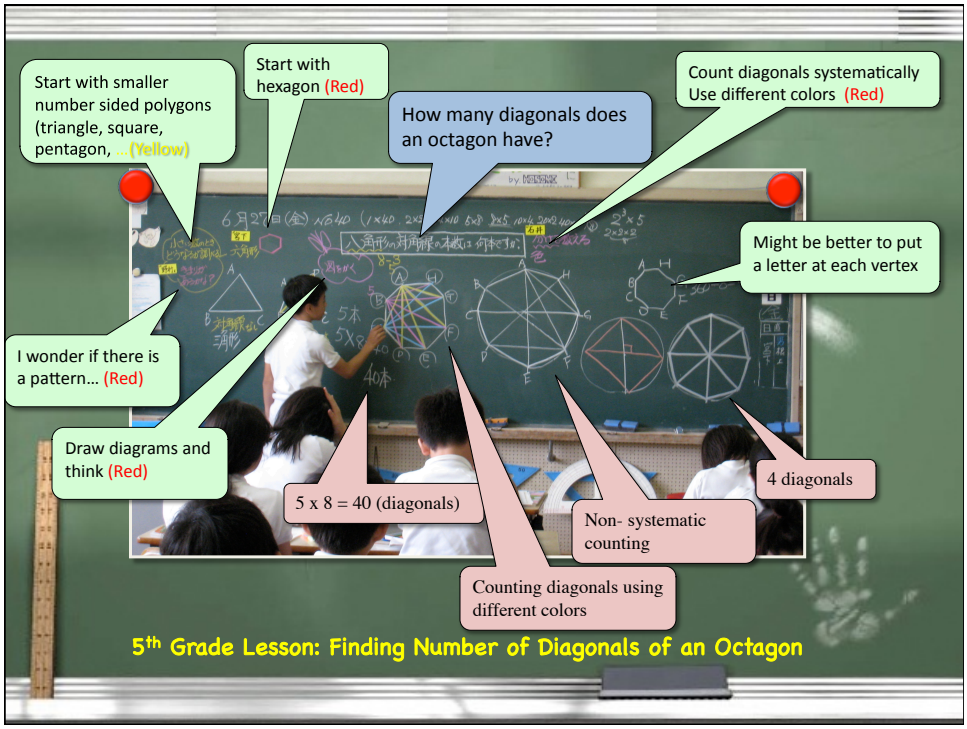
6th Grade Lesson: Developing the Formula for Area of a Parallelogram

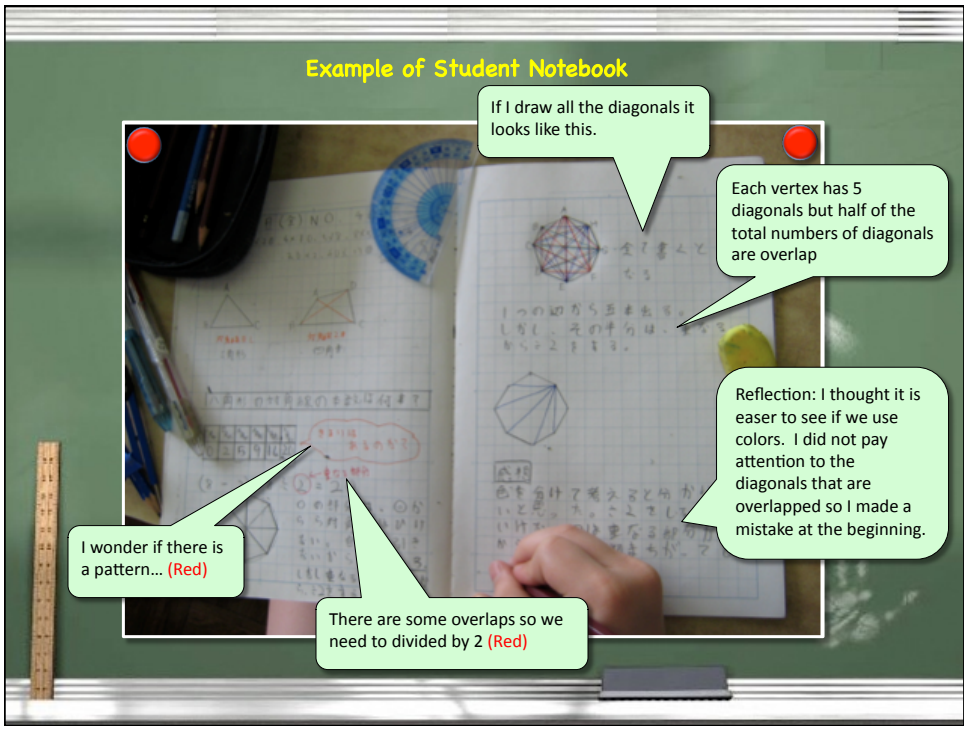
Blackboard Use: Level 4

- Enhances deeper thinking: Compare and discuss Ideas (e.g. mathematical thinking)
- Students' ideas for how to, ways to think, and reflective comments are recorded and discussed,
- Fostering students' organized note-taking skills by modeling good organization (including ideas, method, reflective comments)
- *Bansho* as one large page of notebook that is constructed together with students and teaches as whole class

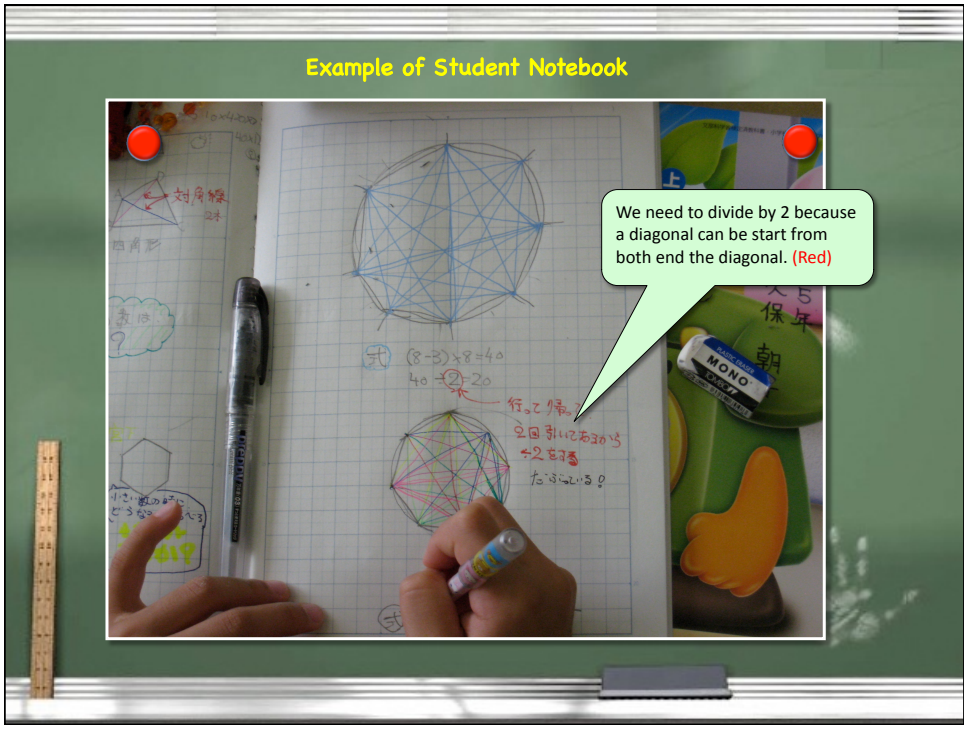
Level of Sophistication of Blackboard Use




Start with smaller number sided polygons (triangle, square, pentagon, ...(Yellow)
Start with hexagon (Red)
How many diagonals does an octagon have?
Count diagonals systematically Use different colors (Red)
Might be better to put a letter at each vertex
I wonder if there is a pattern... (Red)
Draw diagrams and think (Red)
5 x 8 = 40 (diagonals)
4 diagonals
Non- systematic counting
Counting diagonals using different colors
5th Grade Lesson: Finding Number of Diagonals of an Octagon


Example of Student Notebook
If I draw all the diagonals it looks like this.
Each vertex has 5 diagonals but half of the total numbers of diagonals are overlap
Reflection: I thought it is easer to see if we use colors. I did not pay attention to the diagonals that are overlapped so I made a mistake at the beginning.
I wonder if there is a pattern... (Red)
There are some overlaps so we need to divided by 2 (Red)

Example of Student Notebook



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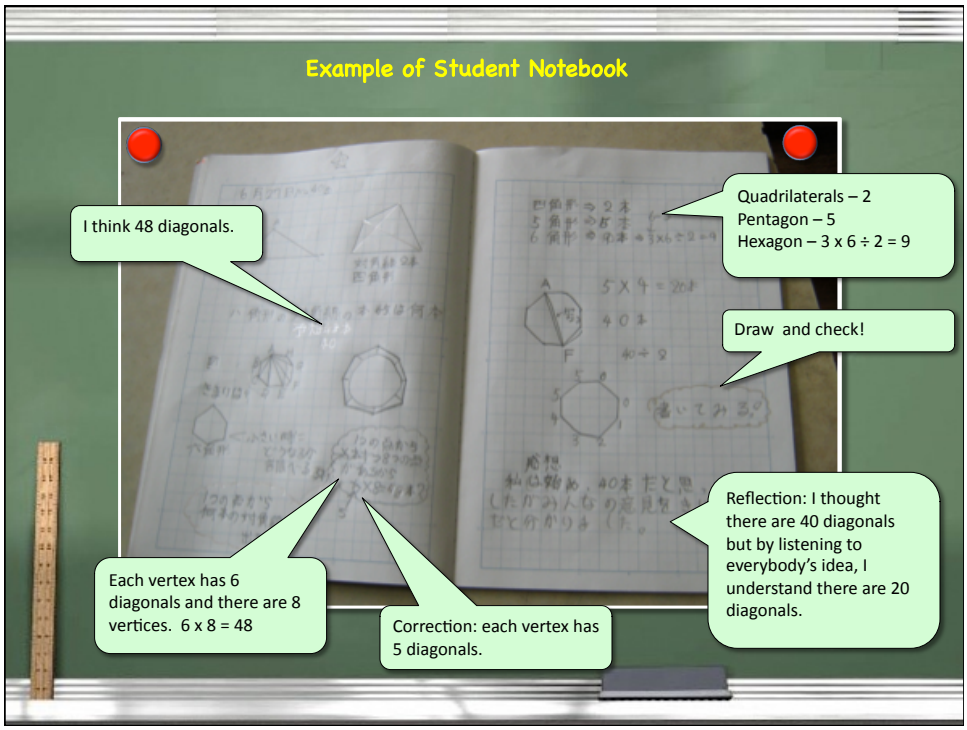

Significance of Bansho

- Help to understand the points of the lesson clearly
- Be able to see and understand the process of the lesson
- Help to promote student thinking
- Be able to gather everybody's ideas
- Be able to compare and contrast ideas and raise the level of thinking and understanding
- Help to think together as a class
- Help to organize and clarify thinking
- Help to improve note taking

Bansho

Keep in mind about:

- Process, flow, and connection of learning
- Goals of the lesson
- Best use of students' thinking, ideas, and voices
- Visually attractive and welcoming
- Developing note taking skills

Strategies for Enhancing the Effectiveness of Blackboards for Improving Student Learning

- Organization of material
  - Sequencing (left to right, top to bottom, chronological)
  - Separate sections
  - Headings (e.g. today's lesson) labels (e.g. student names)
- Differentiation or highlighting of material
  - Use colored chalk
  - Circle or box off parts
  - Student thinking is shown in balloons
  - Teachers versus student manipulatives, diagrams (prepare vs. created)

Strategies for Enhancing the Effectiveness of Blackboards for Improving Student Learning

- Connection of different elements
  - Use of arrows, lines, circle
  - Juxtapose movable materials
  - Use of foldable transformable materials
- Facilitate access
  - Attractive, colorful materials
  - large visuals and print
  - Pre-prepared materials that have been carefully designed to convey key points

Mathematical Instruction

- Three Levels of Mathematical Instruction:
  - **Level 1**: Teacher knows how to perform mathematical procedures and formulas and demonstrate them to the students.
  - **Level 2:** the teacher has conceptual understanding of how and why mathematical procedures and formulas work and tries to explain this understanding to students.
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  (Sugiyama 2008)

Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics

Creating Mathematical Thinkers

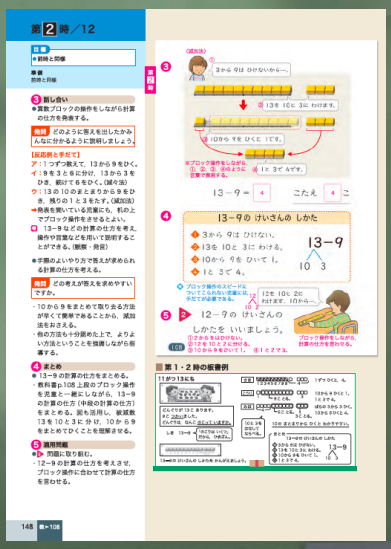
Mathematical Practices

5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make structure
8. Look for and express regularity in repeated reasoning

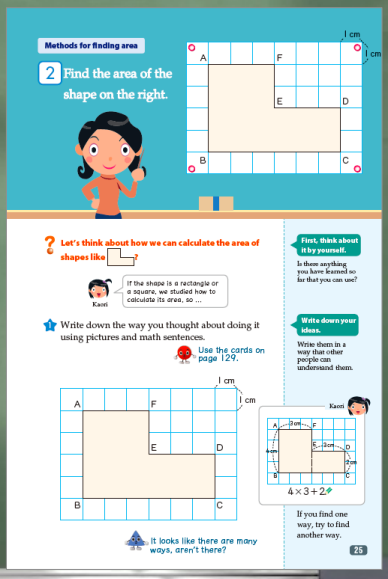
Creating Mathematical Thinkers

- Have Habits of Mind: Creating mathematical thinkers
- Conduct Kyozaikenkyu
- Establish Clear goal(s) of lesson (content, process & proficiency goals.)
- Develop Bansho plan

Resources for Bansho and Note Taking



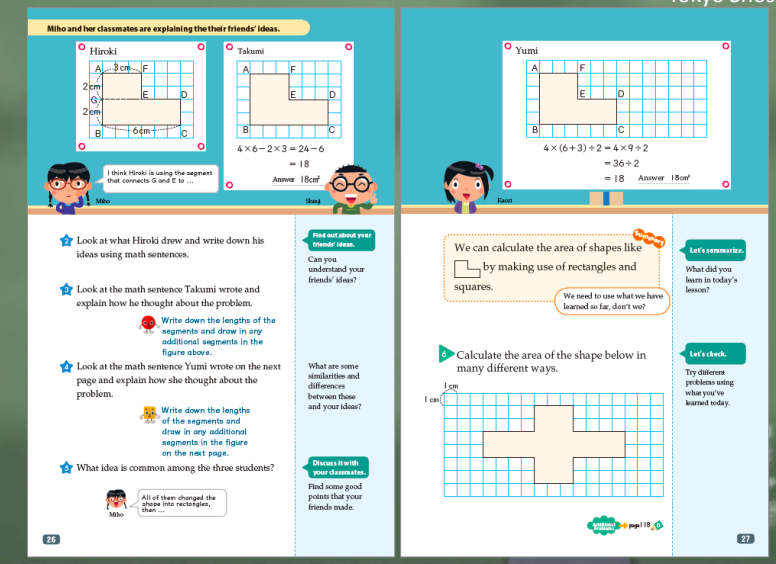
Tokyo Shoseki 2011



Tokyo Shoseki 2011

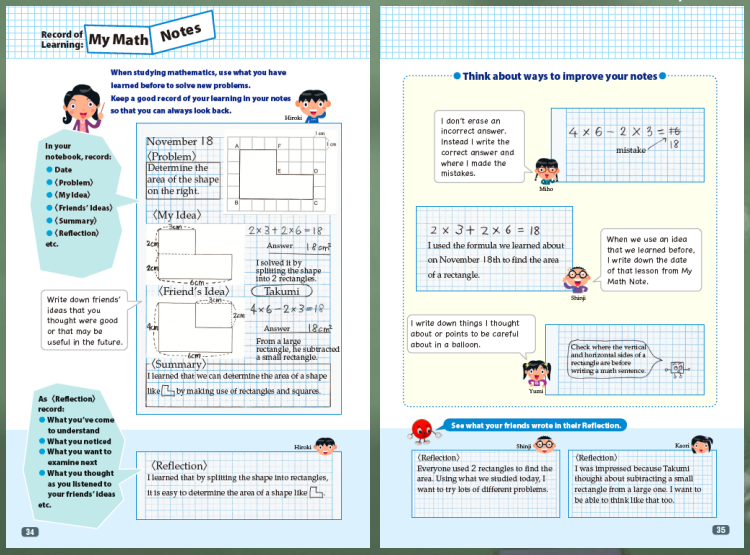
Problem Solving and Board Organization

Tokyo Shoseki 2011



Note Taking

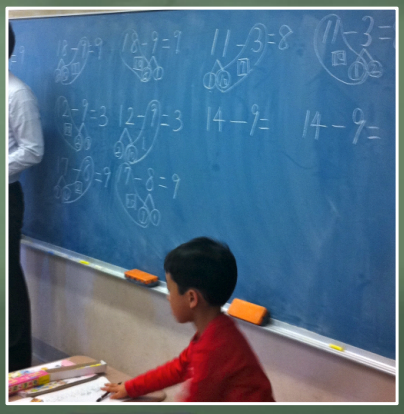
Tokyo Shoseki 2011



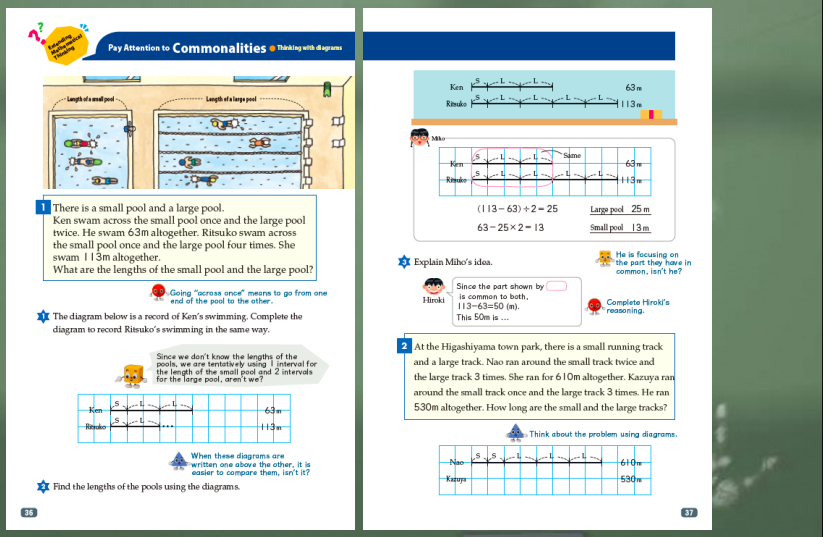
Merits of Note Taking:

- Keeps a record of your own learning (understanding, misunderstanding and mistakes, thinking, progress, strength, weakness, etc.)
- Helpful for reflecting and justifying your own learning
- Helpful resource for problem solving (using prior knowledge to solve a new problem; help to identify what we have learned before and use the information)
- Helpful data for teacher to monitor and evaluate student learning and understanding

Grade 1



Grade 4



Pay Attention to **Commonalities** ● Thinking with diagrams

Length of a small pool — Length of a large pool

1 There is a small pool and a large pool.
Ken swam across the small pool once and the large pool twice. He swam 63m altogether. Ritsuko swam across the small pool once and the large pool four times. She swam 113m altogether.
What are the lengths of the small pool and the large pool?

Going "across once" means to go from one end of the pool to the other.

The diagram below is a record of Ken's swimming. Complete the diagram to record Ritsuko's swimming in the same way.

Since we don't know the lengths of the pools, we are tentatively using 1 interval for the length of the small pool and 2 intervals for the large pool, aren't we?

Ken: S L L — 63m
Ritsuko: S L ... — 113m

When these diagrams are written one above the other, it is easier to compare them, isn't it?

Find the lengths of the pools using the diagrams.

36

Ken: S L L — 63m
Ritsuko: S L L L L — 113m

Miho

Ken: S L L — 63m
Ritsuko: S L L L L — 113m
Same

(113 − 63) ÷ 2 = 25 — Large pool 25 m
63 − 25 × 2 = 13 — Small pool 13 m

Explain Miho's idea.

He is focusing on the part they have in common, isn't he?

Hiroki: Since the part shown by ☐ is common to both, 113−63=50 (m). This 50m is ...

Complete Hiroki's reasoning.

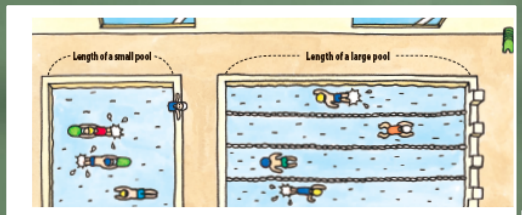
2 At the Higashiyama town park, there is a small running track and a large track. Nao ran around the small track twice and the large track 3 times. She ran for 610m altogether. Kazuya ran around the small track once and the large track 3 times. He ran 530m altogether. How long are the small and the large tracks?

Think about the problem using diagrams.

Nao: S S L L L — 610m
Kazuya: — 530m

37

Grade 4



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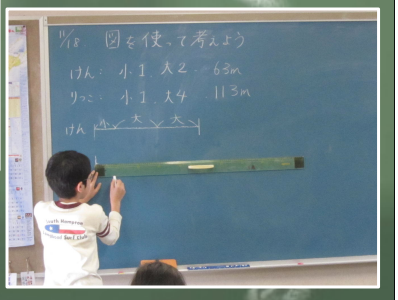
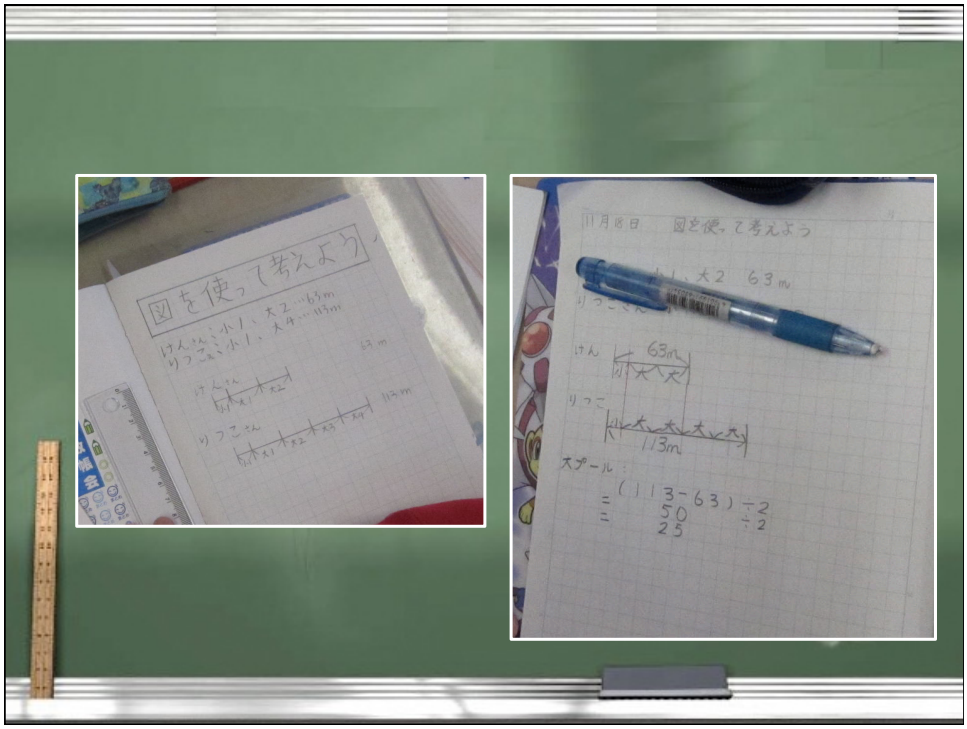
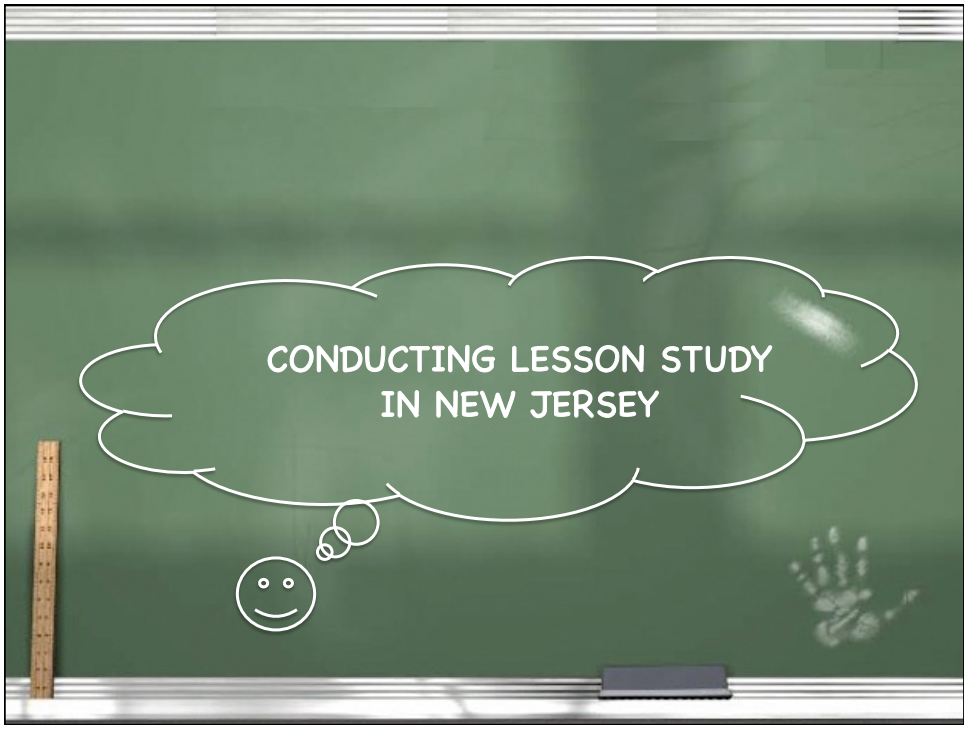



図を使って考えよう
63m
113m
スプール：
(113-63)÷2
50
25


CONDUCTING LESSON STUDY
IN NEW JERSEY

About Paterson Public School #2

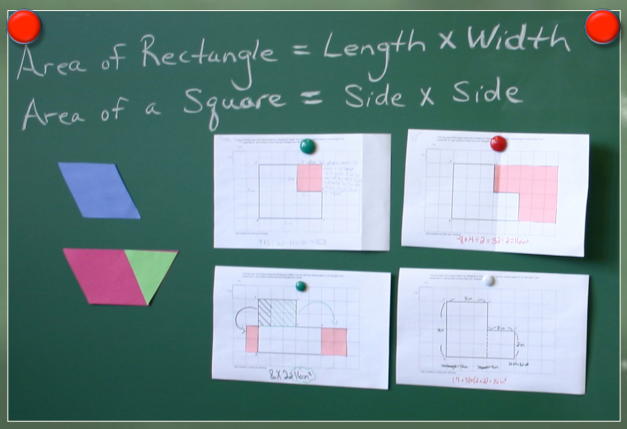
- One of the oldest, continuously-existing public schools in the U.S. (1856).
- Part of the third largest school system in New Jersey
- 720 students in grades pre-K to 8
- 75% Latino, 13% Bengali, 12% African-American
- 98% free lunch
- 30% of students in bilingual programs
- 42% rate of transience

- Lesson Study Goal:

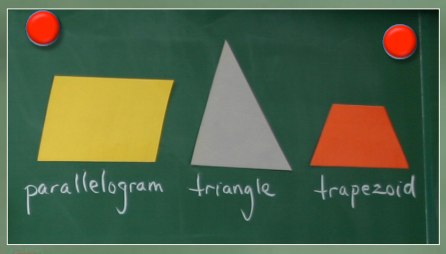
  "Enhancing students' communication skills and mathematical thinking through board organization" (2002 - 2003) (2005 - 2007)

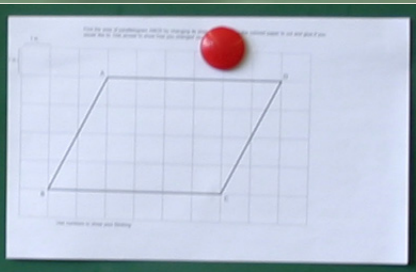
6th Grade Lesson:
Area of Parallelogram (1)
Instructor: Bill Jackson

Review of Previous Lessons



Setting up a Main Problem

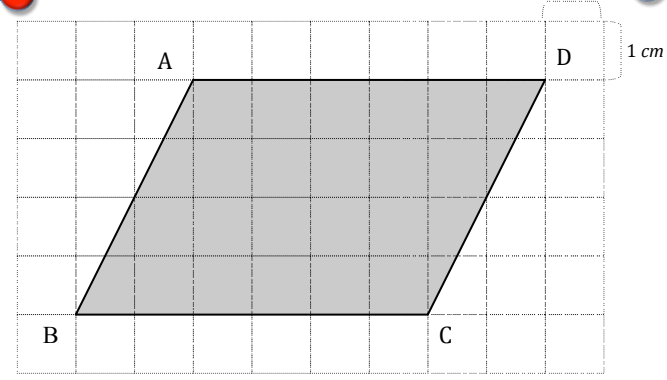




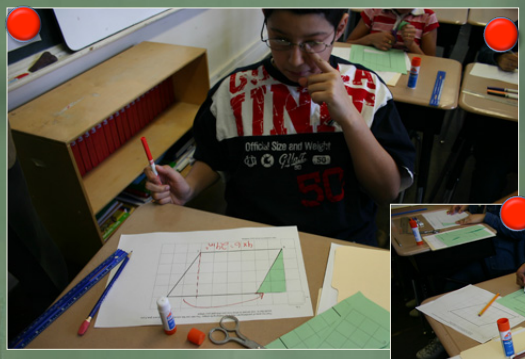
Find the area of the parallelogram by changing its shape.

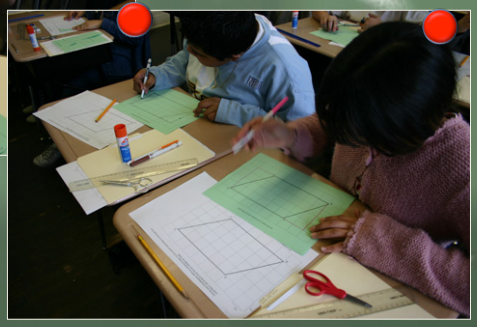
Main Problem

Find the area of the parallelogram by changing its shape.



Students Solving the Problem On Their Own





Student Presentation

Discussing and Clarifying Students' Presentation

Journal Writing

Emmanuel: I learned how to change a shape into a different shape.

Evismari: Today I learned how to turn a shape into another one. It is called transforming a shape.

Jose: Today I learned how to turn a parallelogram into a rectangle.

Jaylene: Today I learned how to transform the shape into a shape I know. I also learned how to find the area.

Roshelle: What I learned today is that you can always find different ways to find the area. You can add, subtract, multiply and divide.

Umme: Today I learned that there are many, many different ways to figure out the area of something. I also learned to use expressions.

Franco: I learned how to transform a shape into an even bigger shape.

Ashley: Today I learned that you can transform the shape or increase the area. Example:

Evi and Umme's method

(6 – 2 + 2) x 4 = 24 sq. units

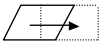
Priscilla's method

(4x2÷2) +(4x4)+(4x2÷2) =24 sq. units.

Roshelle's method

(6 – 1 + 1) x 4 = 24 sq. units

Veronica's method



(6 – 4 + 4) x 4 = 24 sq. units

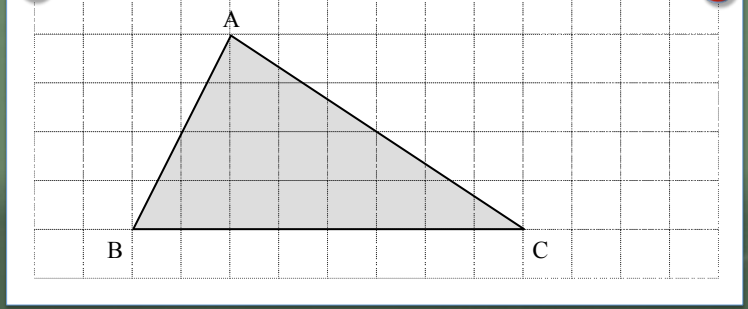
Jose's method

(8x4) – (2x4) = 24 sq. units

6th Grade Lesson: Area of Triangle (1)

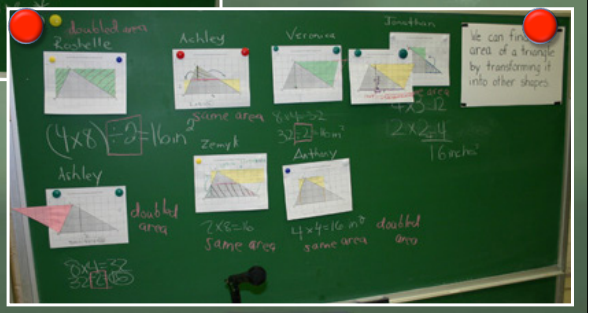
Area of Triangle

Find the area of the triangle by changing its shape.



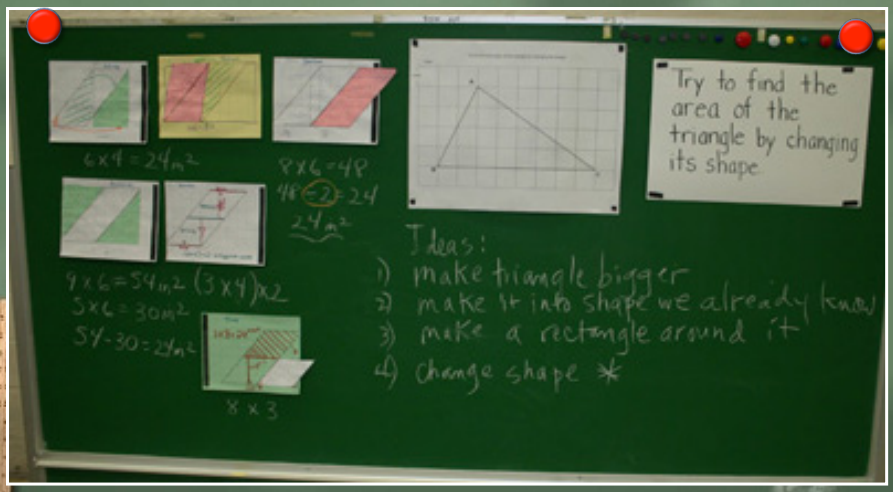
Student Presentation

Blackboard Organization

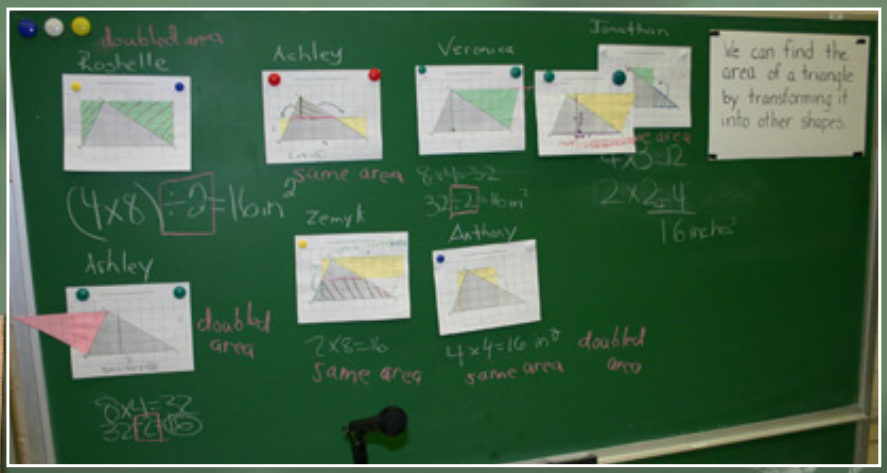


We can find the area of a triangle by transforming it into other shapes.

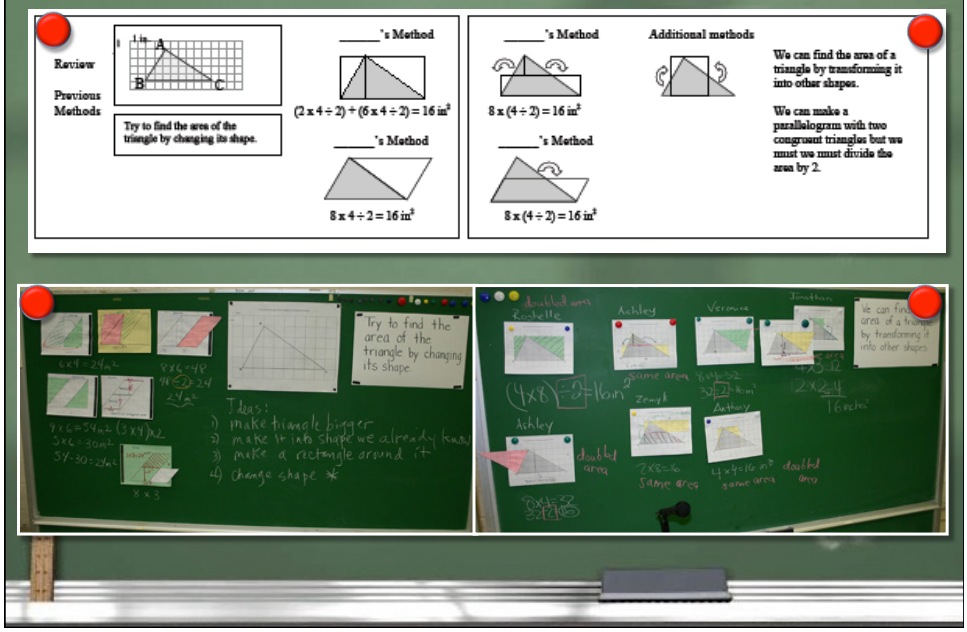
Blackboard Organization



Blackboard Organization



Blackboard Planning

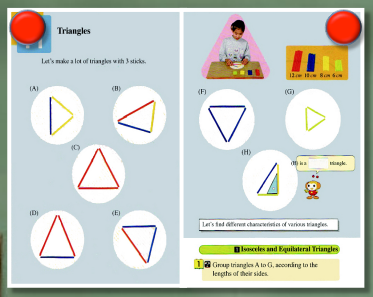


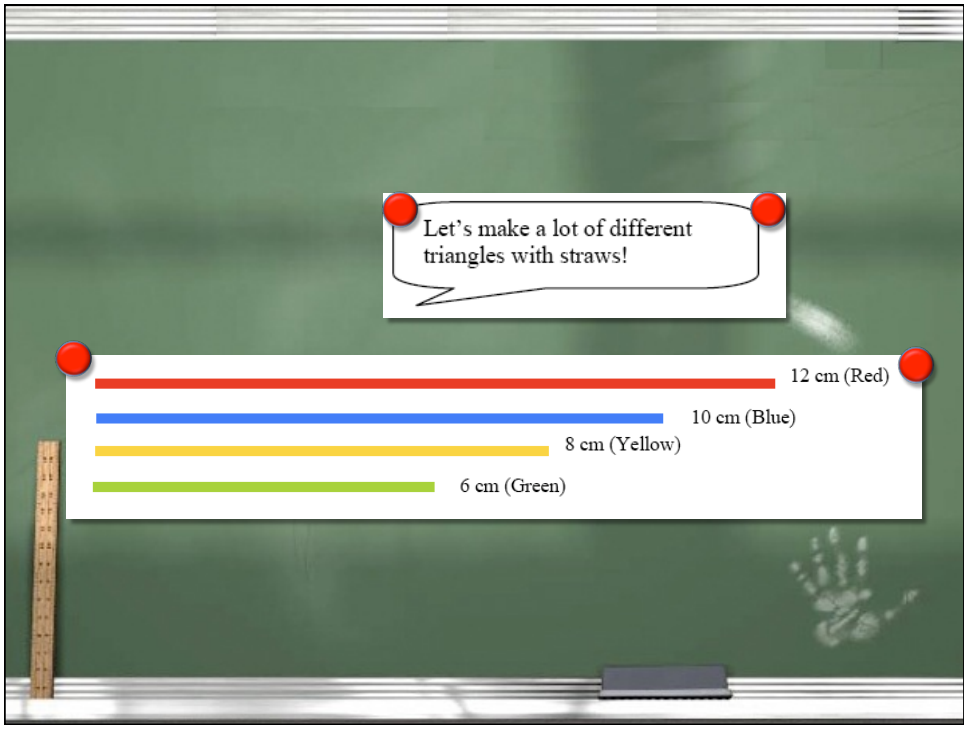
Progression of Blackboard

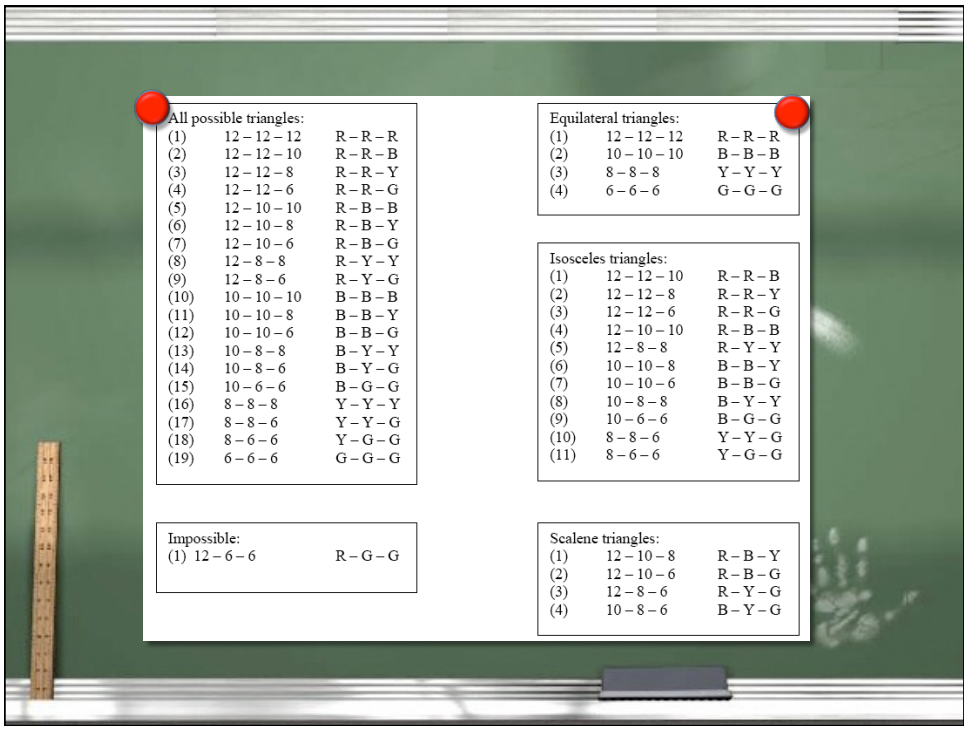
Progression of Blackboard

4th Grade Lesson:
Isosceles and Equilateral Triangles

Kyozaikenkyu on
4th Grade Lesson:
"Isosceles and Equilateral Triangles"







All possible triangles:

(1)	12 – 12 – 12	R – R – R
(2)	12 – 12 – 10	R – R – B
(3)	12 – 12 – 8	R – R – Y
(4)	12 – 12 – 6	R – R – G
(5)	12 – 10 – 10	R – B – B
(6)	12 – 10 – 8	R – B – Y
(7)	12 – 10 – 6	R – B – G
(8)	12 – 8 – 8	R – Y – Y
(9)	12 – 8 – 6	R – Y – G
(10)	10 – 10 – 10	B – B – B
(11)	10 – 10 – 8	B – B – Y
(12)	10 – 10 – 6	B – B – G
(13)	10 – 8 – 8	B – Y – Y
(14)	10 – 8 – 6	B – Y – G
(15)	10 – 6 – 6	B – G – G
(16)	8 – 8 – 8	Y – Y – Y
(17)	8 – 8 – 6	Y – Y – G
(18)	8 – 6 – 6	Y – G – G
(19)	6 – 6 – 6	G – G – G

Impossible:

(1)	12 – 6 – 6	R – G – G

Equilateral triangles:

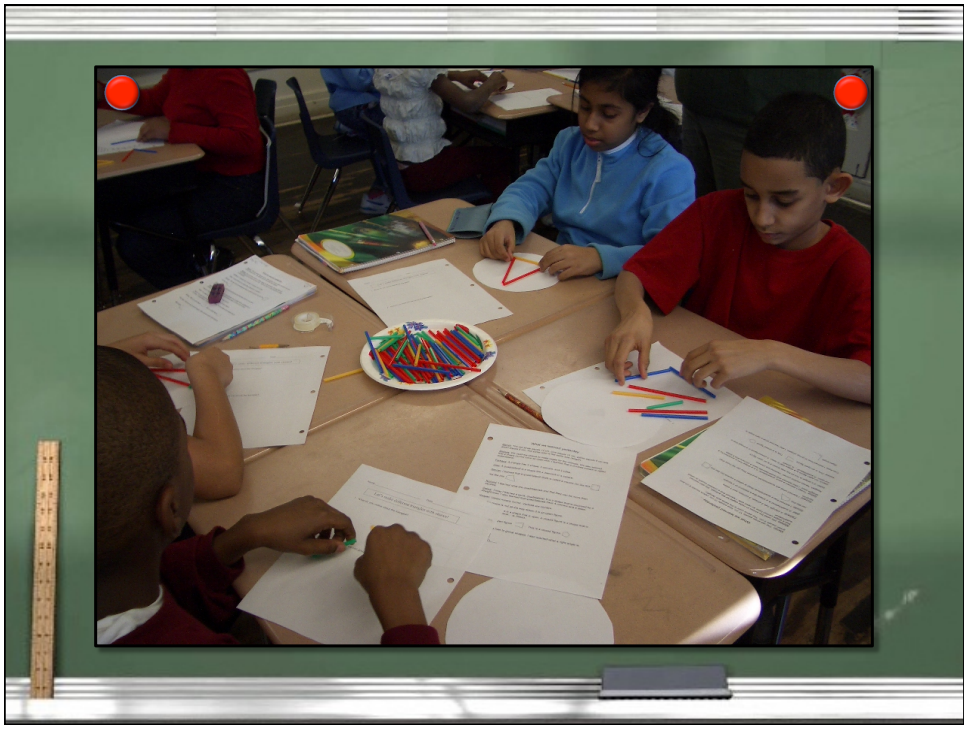
(1)	12 – 12 – 12	R – R – R
(2)	10 – 10 – 10	B – B – B
(3)	8 – 8 – 8	Y – Y – Y
(4)	6 – 6 – 6	G – G – G

Isosceles triangles:

(1)	12 – 12 – 10	R – R – B
(2)	12 – 12 – 8	R – R – Y
(3)	12 – 12 – 6	R – R – G
(4)	12 – 10 – 10	R – B – B
(5)	12 – 8 – 8	R – Y – Y
(6)	10 – 10 – 8	B – B – Y
(7)	10 – 10 – 6	B – B – G
(8)	10 – 8 – 8	B – Y – Y
(9)	10 – 6 – 6	B – G – G
(10)	8 – 8 – 6	Y – Y – G
(11)	8 – 6 – 6	Y – G – G

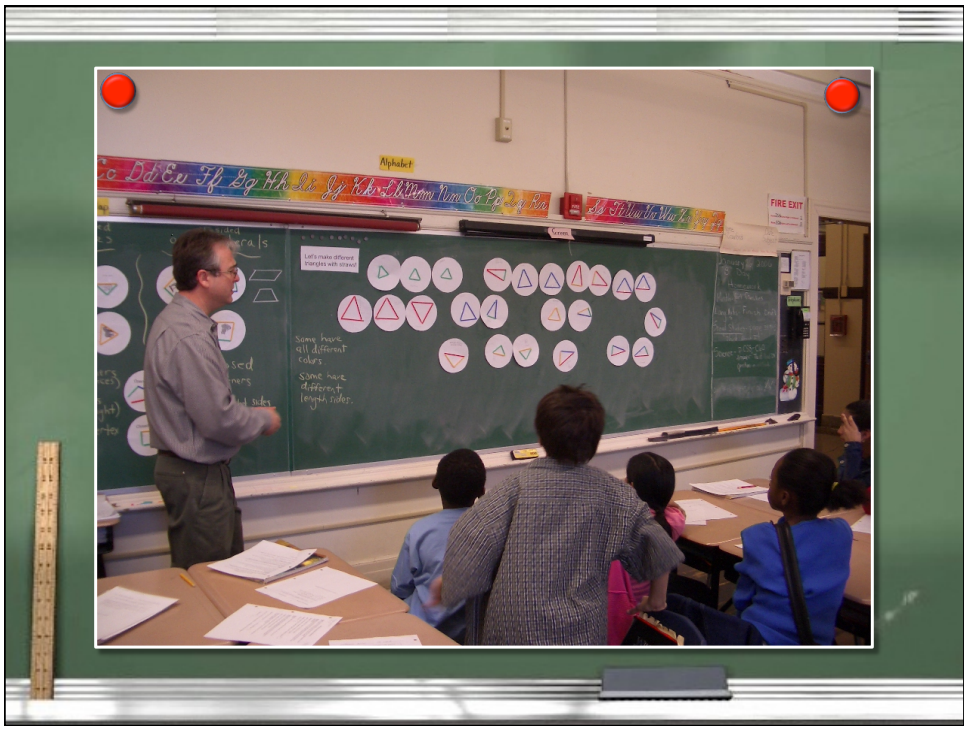
Scalene triangles:

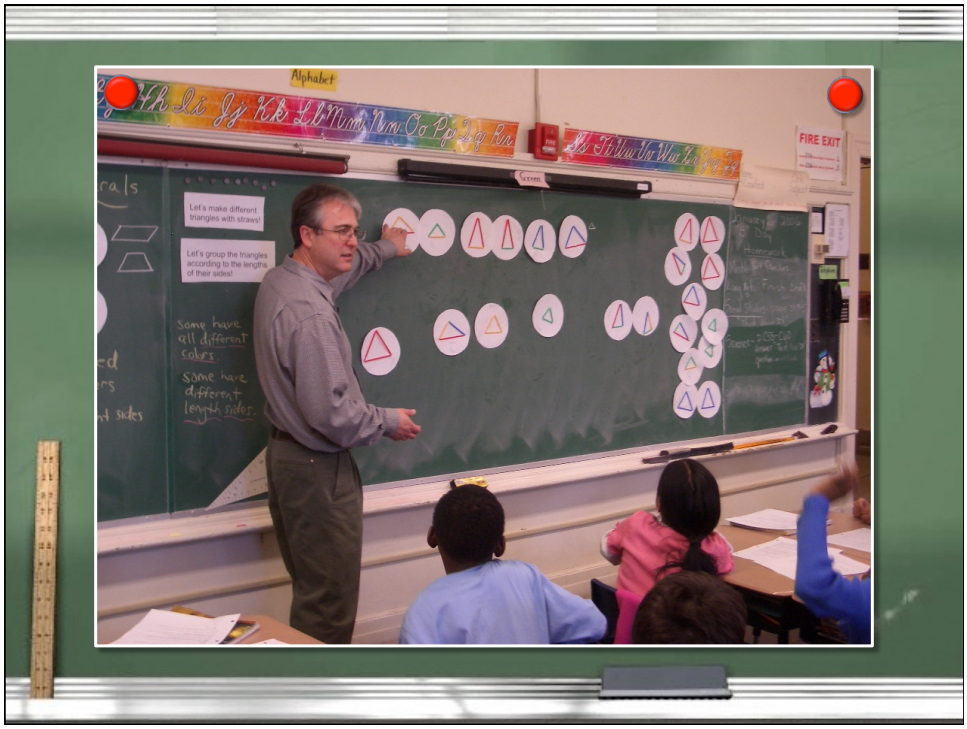
(1)	12 – 10 – 8	R – B – Y
(2)	12 – 10 – 6	R – B – G
(3)	12 – 8 – 6	R – Y – G
(4)	10 – 8 – 6	B – Y – G

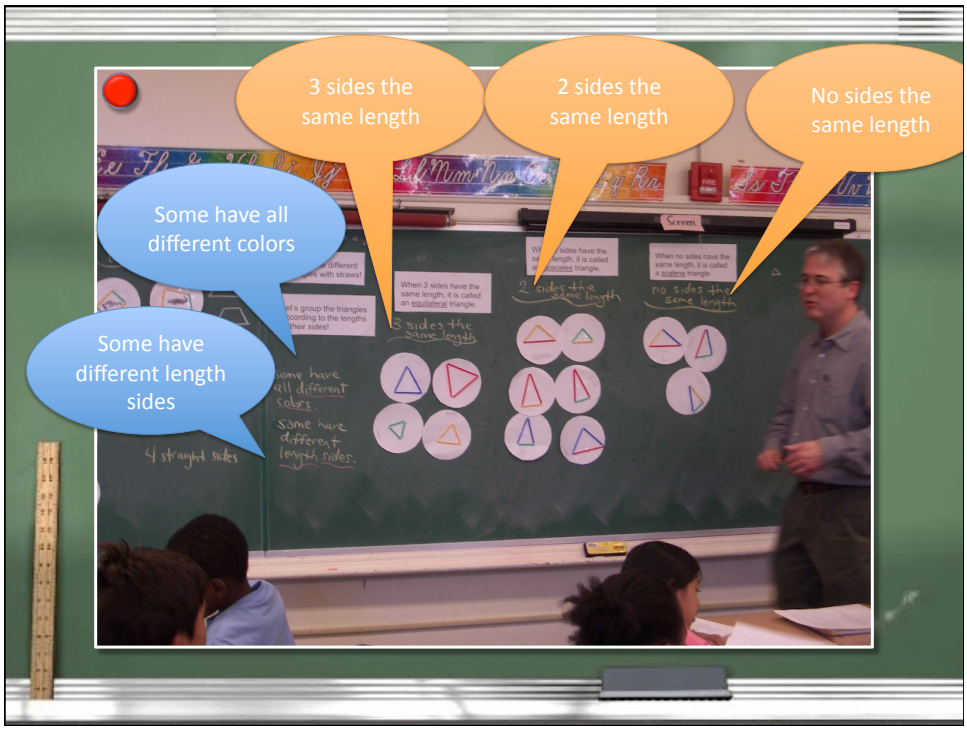


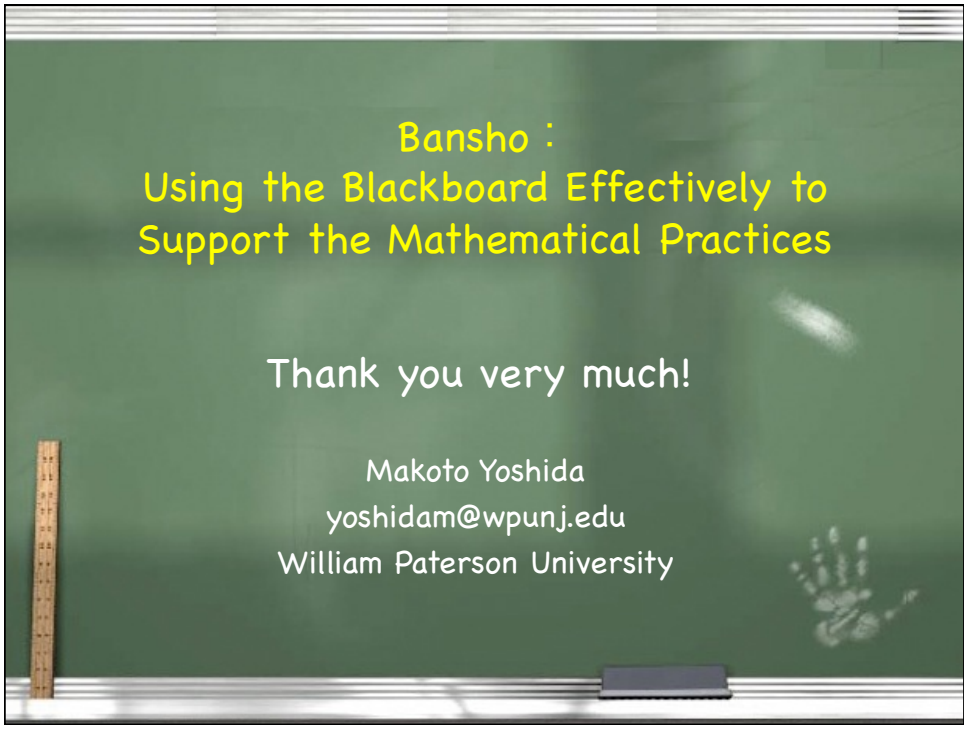

Let's make different triangles with straws!


Alphabet
Screen
Let's make different triangles with straws!


Alphabet
FIRE EXIT
Let's make different triangles with straws!
Some have all different colors
Some have different length sides


Alphabet
FIRE EXIT
Let's make different triangles with straws!
Let's group the triangles according to the lengths of their sides!
Same have all different colors.
Same have different length sides.


3 sides the same length
2 sides the same length
No sides the same length
Some have all different colors
Some have different length sides
Screen
When 3 sides have the same length, it is called an equilateral triangle.
3 sides the same length
2 sides the same length
When no sides have the same length, it is called a scalene triangle.
no sides the same length
Same have all different colors.
Same have different length sides.
4 straight sides


Bansho :
Using the Blackboard Effectively to
Support the Mathematical Practices
Thank you very much!
Makoto Yoshida
yoshidam@wpunj.edu
William Paterson University