Incorporating the Common Core to Your Lesson Study Work

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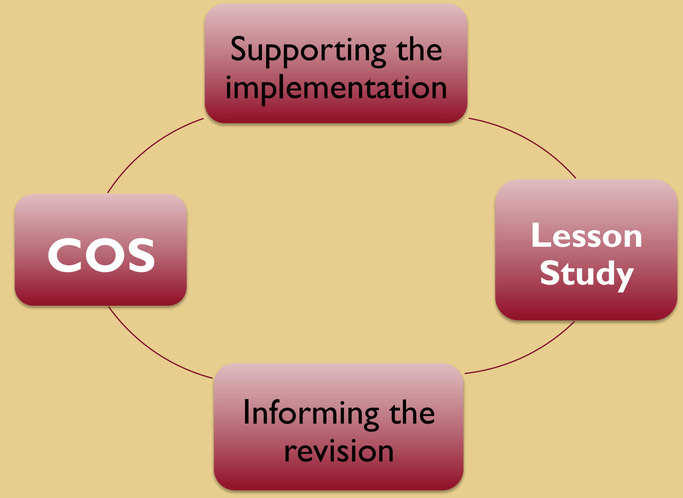


Some common research themes

- How can we teach this **new** topic effectively?
- How can we teach topic X now that it is being discussed a year earlier/later?
  - How can we teach it without topic Y?
  - How can we teach it with additional knowledge of topic Z?
  - How can we teach topic Z so that students can use it effectively in learning topic X?

Some common research themes

- How might we sequence topics in Grade N?
  - Should the study of area formulas in Grade 6 start with (right) triangles, or should it start with parallelograms?
- How might we prepare our students for the implementation in N years?
  - How might we incorporate tape diagrams in the current standards so that students will be able to use them in the future?

Mathematical Practice

- The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.
- These practices rest on important "processes and proficiencies with longstanding importance in mathematics education.

NCTM Process Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

NRC Proficiencies

- Adaptive reasoning
- Strategic competence
- Conceptual understanding
- Procedural fluency
- productive disposition

Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structures.
- Look for and express regularity in repeated reasoning.

Dilemma/Paradox

- Mathematical practice permeates every aspect of mathematics. What and how students learn will be different if a particular content is viewed with mathematical practices in mind.

Dilemma/Paradox

- "If you think about it long enough you can associate just about any practice standard with any content standard, but this sort of matrix thinking can lead to a dilution of the force of the practice standards – if you try to do everything all the time, you end up doing nothing."

(Bill McCallum, March 10, 2011)

Incorporating mathematical practice into your lesson study

- What content standard(s) may be a fruitful location to highlight a particular mathematical practice?
- How might we incorporate (naturally?) a specific mathematical practice in (almost) every lesson?
- Which mathematical practice may be fostered in which part of the unit in focus and how?

More questions

- How might we foster mathematical practice in our students?
  - are there some aspects of standards of mathematical practice that may be best developed through everyday lesson?
  - are there some aspects of standards of mathematical practice that may be more effectively developed through a (series of?) focused lesson(s) on those aspects?

More questions

- How are mathematical practice expectations at different grades different? That is, what might be the developmental trajectory of mathematical practice?

Another way to see MP

Overarching habits of mind of a productive mathematical thinker

1. Make sense of problems and persevere in solving them
6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

Reasoning & Explaining

4. Model with mathematics

5. Use appropriate tools strategically

Modeling and using tools

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Seeing structuers and generalizing

Make sense of problems and persevere in solving them.

- Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.
- They analyze givens, constraints, relationshps and goals.
- They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt.

Make sense of problems and persevere in solving them.

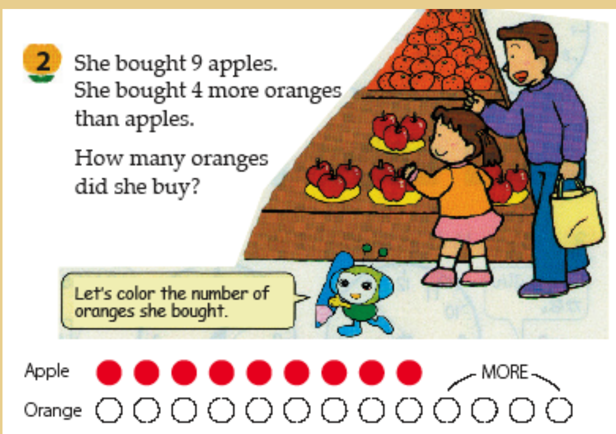
- They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.
- They monitor and evaluate their progress and change course if necessary.
- They can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.

Make sense of problems and persevere in solving them.

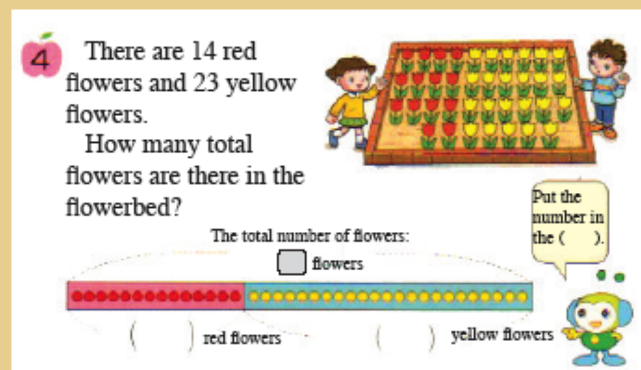
- They check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?"
- They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Reason abstractly and quantitatively

- Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

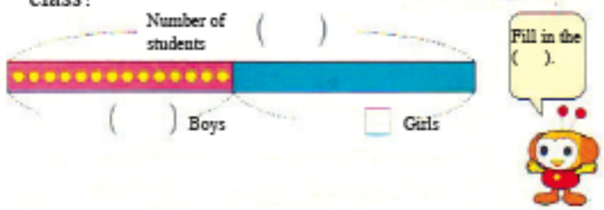


Grade 1, p. 97



Grade 2, A, p. 12

4 In Akira's class, there are 29 total students. There are 14 boys altogether.
How many girls are in his class?



Grade 2, A, p. 20

1 The girl bought a roll of tape. She used 12 *m* of it. She still has 8 *m* of tape left.
How many *m* of the tape did she have at first?

Please fill in the numbers in the diagram below.

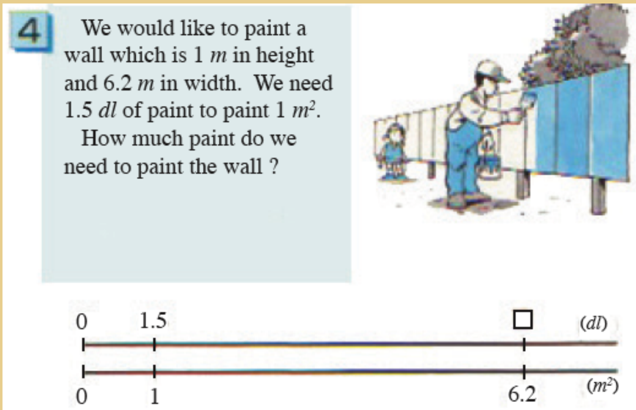
Grade 2, B, p. 52

1 We bought 3 pieces of 32 yen construction paper.
How much did they cost altogether?

Grade 3, A, p. 71

5 1 *m* of an iron bar weighs 1.36 *kg*.
How much does 7 *m* of the same iron bar weigh?

Grade 4, B, p. 64


4 We would like to paint a wall which is 1 m in height and 6.2 m in width. We need 1.5 dl of paint to paint 1 m^2. How much paint do we need to paint the wall ?

Grade 5, B, p. 28

Construct viable arguments and critique the reasoning of others

- Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments.

- 1.G.1: Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
- 2.G.1: Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

- 3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
- 4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

- 8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.
- G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

- G-CO.8 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

Definition of Proof (Stylianides, 2007, p. 291)

Proof is a mathematical argument, a connected sequence of assertions for or against a mathematical claim, with the following characteristics:

1. It uses statements accepted by the classroom community (set of accepted statements) that are true and available without further justification;

Definition of Proof (Stylianides, 2007, p. 291)

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Definition of Proof (Stylianides, 2007, p. 291)

Proof is a mathematical argument, a connected sequence of assertions for or against a mathematical claim, with the following characteristics:

3. It is communicated with forms of expression (modes of argument representation) that are appropriate and known to, or within the conceptual reach of, the classroom community.

3 Dimensions of Development

- Set of mathematical concepts & procedures
- Repertoire of argumentation
- Set of representations

Proof as Explanation

- Using two different digits from 1, 2, 3, …, 9, make a 2-digit number. Reverse the order of digits to make a 2nd 2-digit number.
- Find their sum.
- The sum is always divisible by 11.

Empirical Verification

- 35 + 53 = 88, and 11 divides 88.
- 82 + 28 = 110, and 11 divides 110.
- 97 + 79 = 176, and 11 divides 176.
- etc.

Generic Argument

- Suppose we have 94 and 49.
- If you write the vertically, we have:

$$\begin{array}{r} 9\ 4 \\ +4\ 9 \\ \hline \end{array}$$

- We can switch the ones digits without changing the sum.
- So, we have 99 + 44, and 11 divides both.
- So, the sum must be divisible by 11, too.

Proof

- Let a and b be the two digits.
- The two 2-digit numbers are:

  $10a + b$ and $10b + a$.
- Their sum, therefore, is $10a + b + 10b + a$.
- You can simplify this to $11a + 11b$.
- So, the sum is $11(a + b)$, a multiple of 11.

Model with mathematics

- Mathematically proficient students can aply the mathematics they know to solve problems arising in everyday life, soceity, and the workplace.
- They are comfortable making assumptions and approximations to simplify a complicated situation, realizing that they may need revisionlater.

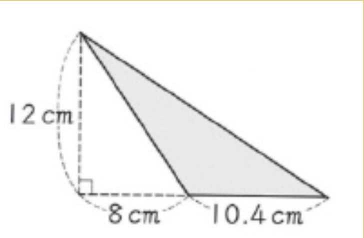
Use appropriate tools strategically

- Mathematically proficient students consider the available tools when solving a mathematical problem.
- They are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.

Attend to precision

- Mathematically proficient students try to communicate precisely to others.
- They try to use clear definitions in discussion with others and in their own reasoning.
- They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.

Find the area.



$$10.4 \times 12 = 124.8 \div 2 = 62.4$$

Look for and make use of structures

- Mathematically proficient students look closely to discern a pattern or structure.
- They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.

Durell & Arnold (1924)

- Some of the principal auxiliary lines used on rectilinear figures are:

1. A line connecting two given points.
2. A line through a given point parallel to a given line.
3. A line through a given point perpendicular to a given line.
4. A line making a given angle with a given line

(p. 103)

Look for and express regularity in repeated reasoning

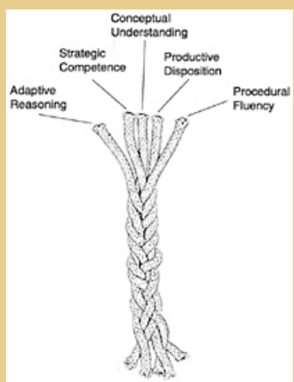
- Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts.
- They maintain oversight of the process, while attending to the details.
- They continually evaluate the reasonableness of their intermediate results.

What are some common "repeated reasoning"?

- You can add/subtract numbers refering to the same unit.
  - 30 + 40 → 3 + 4 units of 10
  - 0.3 + 0.4 → 3 + 4 units of 0.1
  - 3/5 + 4/5 → 3 + 4 units of 1/5
  - 3x + 4x → 3 + 4 units of x
- Deriving area formulas
  - doubling to find the area of triangles
  - doubling to find the area of trapezoids
  - doubling to find the area of kites

Always keep in mind

- Mathematical practice and mathematics content cannot be divorced from each other.

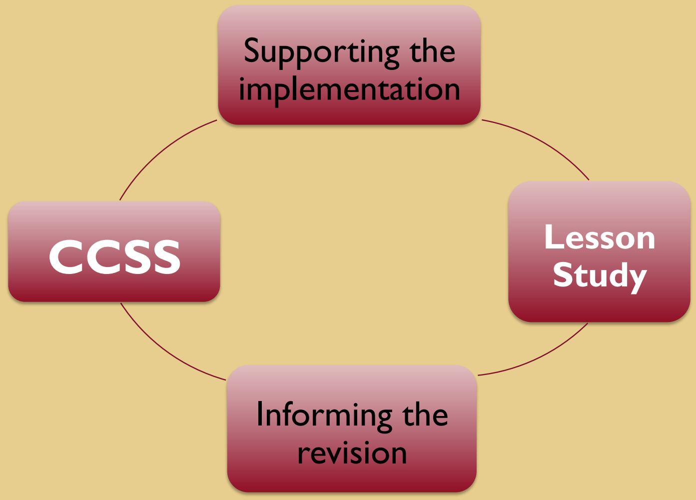


Intertwined Strands of Proficiency

Opportunities

- Common Core State Standards
- Opportunities to share what we learned with our colleagues across the nation.
- Opportunities to collaborate – across grades, across schools, across distrcits, across states.
- Opportunities to make practitioners' knowledge matter more.

Lesson Study & CCSS



Thank you!