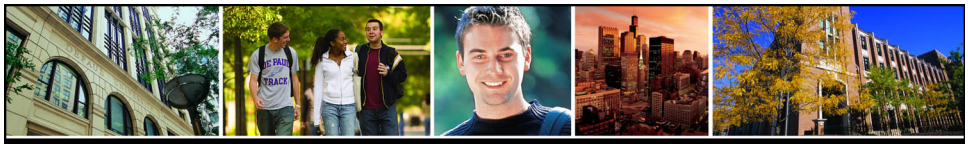

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Teaching through problem solving to develop concepts in the context

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DePaul University

AP≡MSEC
Asia-Pacific Mathematics and Science Education Collaborative

APEC
Asia-Pacific Economic Cooperation

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Purpose of this session

- Teaching though problem solving has been a major focus in mathematics education because it helps students develop concepts in the context so that they can see connections.
- Japanese textbooks use contextualized problems to introduce new concepts to students.
- This session will discuss how we can use problem solving in everyday teaching.

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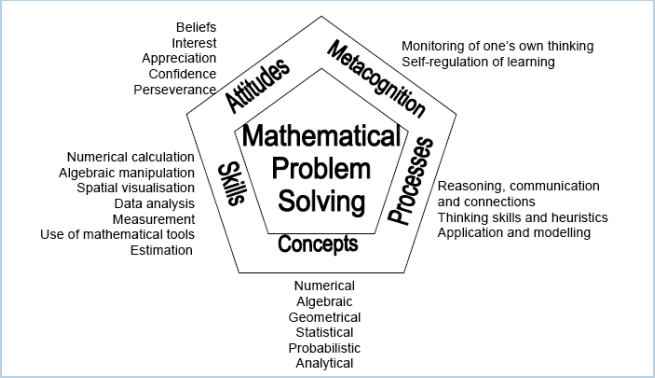
Singapore Syllabus



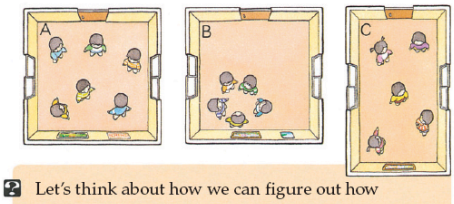
NCTM's view of problem solving

- Problem solving means engaging in a task for which the solution method is not known in advance.
- Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program.
- Choosing worthwhile problems and mathematical tasks
  - There are many, many problems that are interesting and fun but that may not lead to the development of the mathematical ideas that are important for a class at a particular time.

Japanese Math Textbook Grade 5B p.23-25

Crowdedness

1 Kiyoshi and his friends will sleep in cabins A, B and C at camp.
Which cabin is the most crowded?



Let's think about how we can figure out how crowded something is!

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26) The use of "real-world" contexts to introduce mathematical ideas has been advocated, with the term "real world" being used in varied ways. A synthesis of findings from a small number of high-quality studies indicates that if mathematical ideas are taught using "real-world" contexts, then students' performance on assessments involving similar "real-world" problems is improved.
However, performance on assessments more focused on other aspects of mathematics learning, such as computation, simple word problems, and equation solving, is not improved.
(*National Mathematics Advisory Panel FINAL REPORT, 2008)*

Understanding mathematical ideas using "real-world " context

Understanding mathematical ideas beyond the context

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Purpose of the mathematics classroom is to help Students

(Polya, how to solve it)

- One of the most important tasks of the teacher is to help his students.
  - This task is not quite easy; it demands time, practice, devotion, and sound principles.
- The student should acquire as much experience of independent work as possible.
  - But if he is left alone with his problem without any help or with insufficient help, he may make no progress at all.
  - If teacher helps too much, nothing is left to the student.
- The teacher should help, but not too much and not too little, so that student shall have reasonable share of the work.

An Example of a worksheet for "Problem Solving"

- **You are selling ice cream from a cart. You sell ice cream bars for $0.75 per bar. Your cost for the ice cream is $0.30 per bar, and your cost for the rental of the cart is $50.**
  a. In a formula, express your total cost *C* as a function of the number of *n of ice cream bars sold.* On graph paper, graph *C* leaving room for negative values on the y-axis.
  b. Express the revenue *R* generated by the sale of ice cream bars as a function of the number *n* sold. Graph on the same graph as in *a*.
  c. Express the profit *P* generated by the sale of ice cream bars as a function for the number *n* sold. Graph *P* on the same graph as in *a* and *b*.
  d. Fine the break even point graphically and algebraically.

If student is not able to do much
(Polya, how to solve it)

- The teacher should leave him at least some illusion of independent work.
- The best is, however, to help the student naturally.
  - The teacher should put himself in the student's place, he should see the student's case, he should try to understand what is going on in the student's mind, and ask a question or indicate a step that could have occurred to the student himself.

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Japanese Math Textbook Grade 5B p.23-25

Crowdedness

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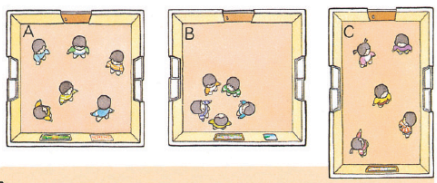

2 Let's think about how we can figure out how crowded something is!

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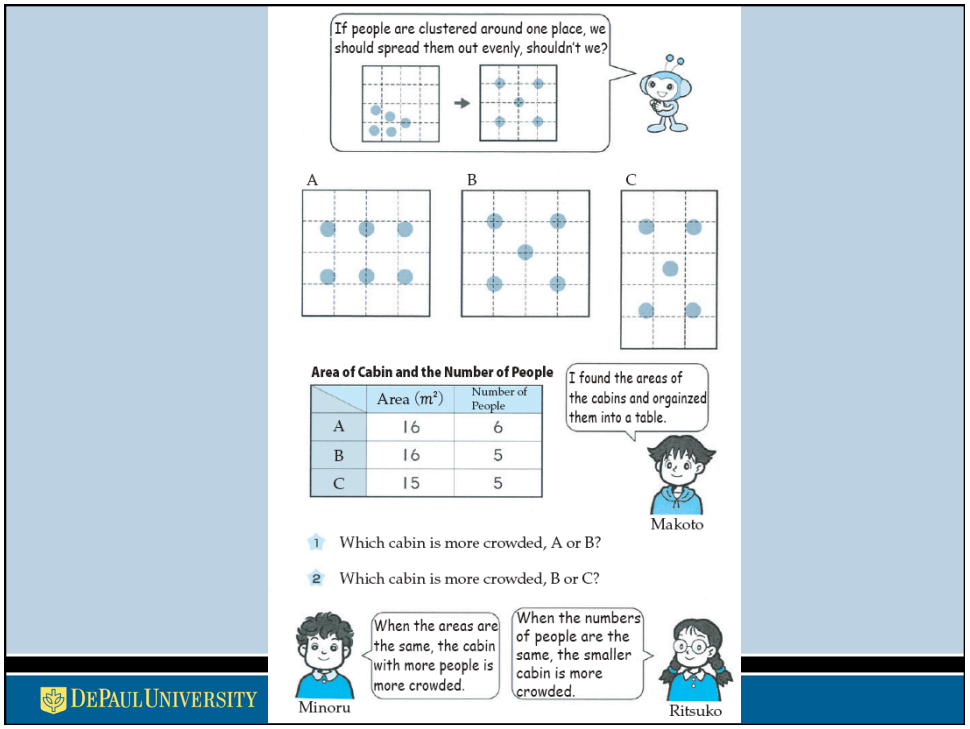

From anticipating students' responses

Area of Cabin and the Number of People

	Area (m^2)	Number of People
A	16	6
B	16	5
C	15	5

Cabin A:16÷6=2.666…
Cabin C: 15÷5=3

m^2 / people

6×5=30
Cabin A:16×5=80
Cabin C: 15×6=90

m^2 / people

Cabin A: 6÷16=0.375
Cabin C: 5÷15=0.333…

people / m^2

16×15=240
Cabin A:6×15=90
Cabin C: 5×16=80

people / m^2

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Using the idea to solve a similar problem (Exercise)

1 Please compare the crowdedness of the gardens of Classes A and B.

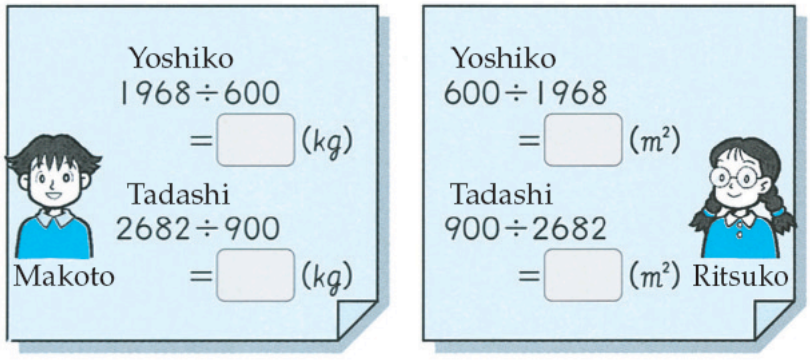
Area of Gardens and Number of Plants

	Area (m^2)	Number of Plants
Class A	15	120
Class B	12	100

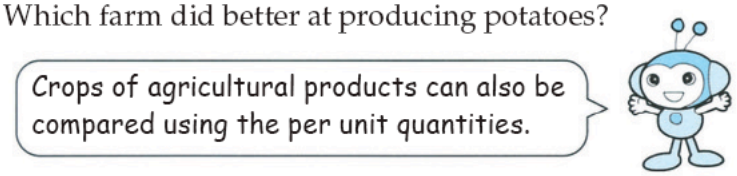
Various per unit quantities

2 At Yoshiko's farm, which is $600m^2$, 1968kg of potatoes were produced.
At Tadashi's farm, which is $900m^2$, 2682kg of potatoes were produced.
Which farm was better at producing potatoes?

1 Please explain these 2 students' methods.



2 Which farm did better at producing potatoes?



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Using the idea to solve a similar problem (Exercise)

1 A car can go 135km on 15ℓ of gasoline. Another car can go 262km on 25ℓ of gasoline.
Which car is more fuel efficient?

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Population density

3 We researched the areas and populations of Toyama city and Ohita city.
Let's compare the crowdedness of Toyama city and of Ohita city!

Area and Populaion of Toyama city and Ohita city (1995)

	Area (km^2)	Population (people)
Toyama city	209	325303
Ohita city	361	426981

1 How many people are there per $1km^2$ in Toyama city and Ohita city ?
Please round your answers to the second highest place.

Toyama city $\cdots 325303 \div 209 =$ ☐ Answer: ☐ people

Ohita city $\cdots 426981 \div 361 =$ ☐ Answer: ☐ people

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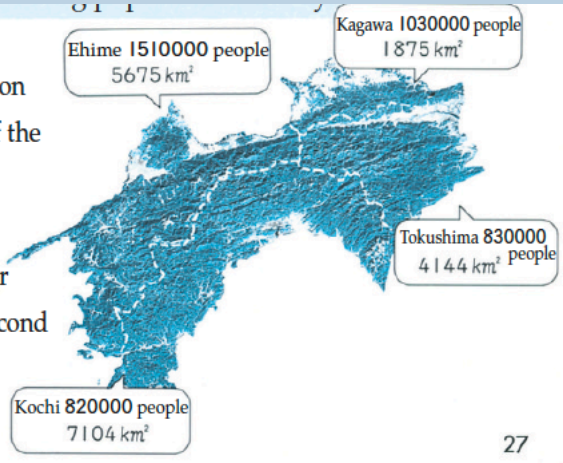
Using the idea to solve a similar problem (Exercise)

1 Find the population density of each of the 4 prefectures in Shikoku.
Please round your answers to the second highest place.



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Practice 1

① Emiko recorded how many minutes she read last week. How many minutes a day did she read on average?

Amount of Time Read

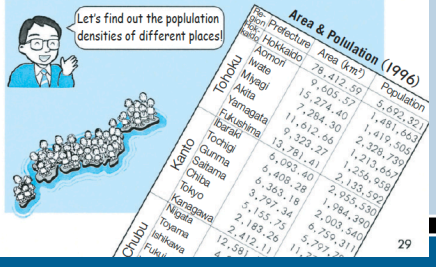
Day of the Week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Time	45min	0	50min	30min	45min	1 h	2 hs

② Five people are in a car. If the average weight of a person is 62kg, and the weight of the car is 1200kg, how many kilograms is the weight of the people and the car altogether?

③ A car can go 7.5km on 1ℓ of gasoline and another car can go 12km on 1ℓ of gasoline.
If the two cars travel the same 270km route, what will be the difference in the amount of gasoline used by the cars?

Challenge

Let's Find Population Densities!



Area & Population (1996)

Region	Prefecture	Area (km²)	Population
Hokkaido	Hokkaido	78,412.59	5,672,321
Tohoku	Aomori	9,605.57	1,481,663
	Iwate	15,274.40	1,416,505
	Miyagi	7,284.30	2,328,739
	Akita	11,612.66	1,213,667
	Yamagata	9,323.27	1,256,958
	Fukushima	13,781.41	2,133,592
Kanto	Ibaraki	6,093.40	2,955,530
	Tochigi	6,408.28	1,984,390
	Gunma	6,363.18	2,003,540
	Saitama	3,797.44	6,759,311
	Chiba	5,155.75	5,797,...
	Tokyo	2,183.26	11,...
	Kanagawa	2,412.11	
Chubu	Niigata	12,581	
	Toyama		
	Ishikawa		
	Fuku...		

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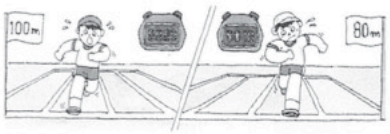
How to compare speed

1 The table on the right shows the distance and time that Susumu and Kiyoshi ran.
Who ran faster, Susumu or Kiyoshi?

Distance and Time

	Distance (m)	Time (sec.)
Susumu	100	20
Kiyoshi	80	18



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How to express speed

2 The bullet train "Hikari" travels 558km in 3 hours, and "Yamabiko" travels 392km in 2 hours. Which travels faster?

Let's think about how we can express speed!

Three Levels of Teaching

- Level 1: Teachers can tell students important basic ideas of mathematics such as facts, concepts, and procedures.
- Level 2: Teachers can explain the meanings and reasons of the important basic ideas of mathematics in order for students to understand them.
- Level 3: Teachers can provide students opportunities to understand these basic ideas, and support their learning so that the students become independent learners.

In order to develop expertise…..

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- learning by reading, listening, and seeing is not sufficient.
- It also requires learning through planning, doing, and reflecting.

Two major types of professional development

- Phase 1 professional development focuses on developing the knowledge for teaching mathematics,
  - through reading books and resources, listening to lectures, and watching visual resources such and video and demonstration lessons.
- Phase 2 professional development focuses on developing expertise for teaching mathematics
  - teachers should plan the lesson carefully, teach the lesson based on the lesson plan, and reflect upon the teaching and learning based on the careful observation. Japanese teachers and educators usually go through this process using Lesson Study

Two major types of professional development programs

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learning by reading, listening and seeing

learning through planning, doing, and reflecting

Lesson Study is an ideal Phase 2 Professional Development

Traditional Workshop

- Begins with answer
- Driven by outside "expert"
- Communication flow: trainer to teachers
- Hierarchical relations between trainer & learners
- Research informs practice

Lesson Study

- Begins with question
- Driven by participants
- Communication flow: among teachers
- Reciprocal relations among learners
- Practice is research

Contrasting methods of professional development (reprinted from Lewis, 2002, p.12)

Japanese Math Textbook Grade 1 p .32